Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Given that the equation $z^2 + (p + iq)z + r + is = 0$ has a real root where p, q, r, s $\in$ R. Then which one is correct-

(A) $pqr = r^2 + p^2 s$ (B) $prs = q^2 - r^2 p$
(C) $qrs = p^2 + s^2 q$ (D) $pqs = s^2 + q^2 r$

Q.2 If $(x + iy)(p + iq) = (x^2 + y^2)i$, then-

(A) $p = x, q = y$ (B) $p = x^2, q = y^2$
(C) $x = q, y = p$ (D) None of these

Q.3 If ω is one of the imaginary cube root of unity, then the value of the expression $(1 + 2\omega + 2\omega^2)^{10} + (2 + \omega + 2\omega^2)^{10} + (2 + 2\omega + \omega^2)^{10}$ is-

(A) 0 (B) 1
(C) ω (D) ω^2

Q.4 If $\frac{3 + 2i\sin x}{1 - 2i\sin x}$ is purely imaginary then x =

(A) $n\pi \pm \frac{\pi}{6}$ (B) $n\pi \pm \frac{\pi}{3}$
(C) $2n\pi \pm \frac{\pi}{3}$ (D) $2n\pi \pm \frac{\pi}{6}$

Q.5 The sequence $s = i + 2i^2 + 3i^3 + \ldots\ldots\ldots$ upto 100 terms simplifies to where $i = \sqrt{-1}$

(A) $50(1 - i)$ (B) $25i$
(C) $25(1 + i)$ (D) $100(1 - i)$

Q.6 If $z + z^3 = 0$ then which of the following must be true on the complex plane-

(A) $Re(z) < 0$ (B) $Re(z) = 0$
(C) $Im(z) = 0$ (D) $z^4 = 1$

Q.7 The number of solutions of the equation $z^2 + z = 0$, where z is a complex number is-

(A) 4 (B) 3
(C) 2 (D) 1

Q.8 Square root of $x^2 + \frac{1}{x^2} - \frac{4}{i}\left(x - \frac{1}{x}\right) - 6$ where x $\in$ R is equal to-

(A) $\pm\left(x - \frac{1}{x} + 2i\right)$ (B) $\pm\left(x - \frac{1}{x} - 2i\right)$
(C) $\pm\left(x + \frac{1}{x} + 2i\right)$ (D) $\pm\left(x + \frac{1}{x} - 2i\right)$

Q.9 The complex number z satisfies $z + |z| = 2 + 8i$. The value of $|z|$ is -

(A) 10 (B) 13
(C) 17 (D) 23

Q.10 A point z moves on the curve $|z - 4 - 3i| = 2$ in an argand plane. The maximum & minimum values of $|z|$ are-

(A) 2, 1 (B) 6, 5
(C) 4, 3 (D) 7, 3

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 3) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○
10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	C	A	A	A	B	C	A	C	D

SOLUTIONS

Sol.1 (D)

$z^2 + (p + iq)z + r + i\,s = 0$

Let $z = x + iy$

$\Rightarrow (x + iy)^2 + (p + iq)(x + iy) + r + i\,s = 0$

$\Rightarrow (x^2 - y^2 + 2ixy) + (px + ipy + iqx - qy) + r + i\,s = 0$

$\Rightarrow (x^2 - y^2 + px - qy + r) + i\,(2xy + py + qx + s) = 0 + i0$

$\Rightarrow x^2 - y^2 + px - qy + r = 0$ .... (1)

and $2xy + py + qx + s = 0$ .... (2)

If the roots are real $y = 0$ this implies (1) & (2) becomes

$\Rightarrow x^2 + px + r = 0$ and $qx + s = 0$

$\Rightarrow x = -\dfrac{s}{q}$

$\therefore \dfrac{s^2}{q^2} - \dfrac{sp}{q} + r = 0$

$\Rightarrow s^2 - spq + rq^2 = 0$

$\Rightarrow pqs = s^2 + rq^2$

Sol.2 (C)

$(x + iy)(p + iq) = (x^2 + y^2)\,i$

$\Rightarrow i(x + iy)(p + iq) = -(x^2 + y^2)$ $\quad \because i^2 = -1$

$(ix - y)(p + iq) = -(x^2 + y^2)$

$\Rightarrow (-y + ix)(p + iq) = -(x^2 + y^2)$

$\Rightarrow -yp - iyq + ixp - xq = -(x^2 + y^2)$

Hence L.H.S. = R.H.S. if $y = p$ and $x = q$

Sol.3 (A)

$(1 + 2(\omega + \omega^2))^{10} + (2(1 + \omega^2) + \omega)^{10} + (2(1 + \omega) + \omega^2)^{10}$

$= (1 + 2(-1))^{10} + (2(-\omega) + \omega)^{10} + (2(-\omega^2) + \omega^2)^{10}$

$= (-1)^{10} + (-\omega)^{10} + (-\omega^2)^{10}$

$= 1 + \omega + \omega^2 = 0$

Sol.4 (A)

Let $z = \dfrac{3 + 2i\sin x}{1 - 2i\sin x}$

$= \dfrac{3 + 2i\sin x}{1 - 2i\sin x} \times \dfrac{3 + 2i\sin x}{1 + 2i\sin x}$

$= \dfrac{3 + 8i\sin x - 4\sin^2 x}{1 + 4\sin^2 x}$

$z = \dfrac{(3 - 4\sin^2 x) + 8i\sin x}{1 + 4\sin^2 x}$

$\therefore$ z is purely imaginary if $\text{Re}(z) = 0$

$\Rightarrow 3 - 4\sin^2 x = 0 \quad \sin^2 x = \dfrac{3}{4}$

$\Rightarrow \sin x = \pm \dfrac{\sqrt{3}}{2}$

$\Rightarrow x = n\pi \pm \dfrac{\pi}{6}$

Sol.5 (A)

$S = i + 2i^2 + 3i^3 + \ldots\ldots\ldots\ldots$ 100 terms ...... (1)

$Si = i^2 + 2i^3 + \ldots\ldots\ldots + T_{99}\, i + T_{100}\, i$ ...... (2)

Subtracting (2) from (1), we get

$S(1 - i) = (i + i^2 + i^3 + \ldots\ldots\ldots + T_{100} - T_{99}\, i) - T_{100}\, i$

$S(1 - i) = (i + i^2 + i^3 + \ldots\ldots\ldots + T_{100}$ terms$) - 100i$ $\quad \because T_{100} = 100$

$\Rightarrow S(1 - i) = \dfrac{i(i^{100} - 1)}{i - 1} - 100i$

$\Rightarrow S(1 - i) = 100i$

$\Rightarrow S = \dfrac{100i}{1 - i} \times \dfrac{1 + i}{1 + i}$

$S = 50(1 - i)$

Sol.6 (B)

$z + z^3 = 0$

$z(1 + z^2) = 0$

$z = 0$ or $z^2 = -1$

$\Rightarrow z = i$

$\Rightarrow z = 0$ or $z = i$

$\Rightarrow x = 0$ i.e. $\text{Re}|z| = 0$.

Sol.7 (C)

$z^2 + z = 0$

$(x + iy)^2 + x + iy = 0$

$x^2 - y^2 + 2ixy + x + iy = 0$

$x^2 - y^2 + x + i\,(2xy + y) = 0$

$\Rightarrow\ x^2 - y^2 + x = 0$ and $2xy + y = 0$

Now $2xy + y = 0$ gives $y = 0$ or $x = -\frac{1}{2}$

I. When $y = 0$

$x^2 - y^2 + x = 0$

gives $x^2 + x = 0$, $x = 0, -1$

II. When $x = -\frac{1}{2}$,

$x^2 + x - y^2 = 0$ gives

$\frac{1}{4} - \frac{1}{2} - y^2 = 0$

$\Rightarrow\ -\frac{1}{4} = y^2$

$\Rightarrow\ y = \pm\frac{1}{2}i$ not possible.

$\Rightarrow$ Only two solutions z(0, 0) and (– 1, 0)

Sol.8 (A)

$x^2 + \frac{1}{x^2} - \frac{4}{i}\left(x - \frac{1}{x}\right) - 6$

$\Rightarrow\ \left(x - \frac{1}{x}\right)^2 - \frac{4}{i}\left(x - \frac{1}{x}\right) - 4$

$= \left(x - \frac{1}{x}\right)^2 + 4i\left(x - \frac{1}{x}\right) - 4$

Let $x - \frac{1}{x} = y$

$\Rightarrow\ y^2 + 4iy - 4$

$\Rightarrow\ y = -\frac{-4i \pm \sqrt{(4i)^2 - 4(-4)1}}{2}$

$= \frac{-4i \pm \sqrt{-16 + 16}}{2} = -2i$

$\therefore\ y = -2i$

$\Rightarrow\ x - \frac{1}{x} = -2i$

$\Rightarrow\ \left(x - \frac{1}{x} + 2i\right) = 0$ is one of the root.

Sol.9 (C)

$z + |z| = 2 + 8i$

$\Rightarrow\ x + iy + \sqrt{x^2 + y^2} = 2 + 8i$

$\Rightarrow\ y = 8$ & $x + \sqrt{x^2 + y^2} = 2$

$\Rightarrow\ x + \sqrt{x^2 + 64} = 2$

$\Rightarrow\ x^2 + 64 = (2 - x)^2$

$\Rightarrow\ 64 = -4x + 4$

$\Rightarrow\ 60 = -4x$

$\Rightarrow\ x = -15$

$\therefore\ |z| = \sqrt{x^2 + y^2} = \sqrt{225 + 64}$

$= \sqrt{289} = 17$

Sol.10 (D)

$|z - 4 - 3i| = 2$

Now $|z| = |z - 4 - 3i + 4 + 3i|$

$= |(z - 4 - 3i) + (4 + 3i)|$

$\le |z - 4 - 3i| + |4 - 3i|$

$\le 2 + \sqrt{16 + 9}$

$\le 2 + 5$

≤ 7

Again $|z| = |(z - 4 - 3i) + (4 + 3i)|$

$= |(4 + 3i) - (-z + 4 + 3i)|$

$\ge |4 + 3i| - |-z + 4 + 3i|$

$\ge |4 + 3i| - |z - 4 - 3i|$

$\because\ |z| = |-z|$

$\ge 5 - 2$

≥ 3

$\Rightarrow$ Max. value and min value of $|z|$ are 7, 3