Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-8(+3,–1). (All questions have only one option correct).

Q.1 Two cars are travelling along two roads which cross each other at right angles at A. One car is travelling towards A at 21 miles per hour while the other is travelling towards it at 28 miles per hour. If initially their distance from A are 1500 feet and 2100 feet respectively, the least distance between them is-
(A) 60 ft (B) 40 ft (C) 20 ft (D) None

Q.2 The set of all values of k for which the function $f(x) = (k^2 - 3k + 2)\left(\cos^2\frac{x}{4} - \sin^2\frac{x}{4}\right) + (k-1)x + \sin 1$ does not possess critical points is-
(A) $[1, \infty)$ (B) $(0, 1) \cup (1, 4)$
(C) $(-2, 4)$ (D) $(1, 3) \cup (3, 5)$

Q.3 Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by 200 students is to be as small as possible, then the school should be built at-
(A) Town B
(B) 45 km. from town A
(C) Town A
(D) 45 km from town B

Q.4 In the curve $x = a[\cos t + \log \tan (t/2)]$, $y = a \sin t$, the portion of the tangent between the point of contact and the x-axis is of length-
(A) 2a (B) a
(C) a/2 (D) None

Q.5 If tangent to the curve $x = at^2$, $y = 2at$ is perpendicular to x-axis then its point of contact is-
(A) (a, a) (B) (0, a)
(C) (a, 0) (D) (0, 0)

PASSAGE :

A window of fixed perimeter (including the base of the arch) is in the form of a rectangle surmounted by a semi-circle. The semi-circular portion is fitted with coloured glass, while the rectangular portion is fitted with clear glass. The clear glass transmits three times as much light per square metre as the coloured glass.
Suppose that y is the length and x is the breadth of the rectangular portion and p the perimeter.

Q.6 The ratio of the sides y : x of the rectangle so that the window transmit the maximum light is-
(A) 3 : 2 (B) $6 : 6 + \pi$
(C) $6 + \pi : 6$ (D) 1 : 2

Q.7 If μ is the amount of light per square metre for the coloured glass and L is the total light transmitted then $\left.\frac{dL}{dy}\right|_{y=1}$ is equal to-

(A) $\frac{\mu}{2}\left[3p - 12 - \frac{5\pi}{2}\right]$ (B) $\mu\left[2p - 6 - \frac{3\pi}{2}\right]$

(C) $\mu\left[p - 2 - \frac{\pi}{2}\right]$ (D) $2\mu\left[3p - \frac{\pi}{2}\right]$

Q.8 Critical points of L are-

(A) $y = \frac{p}{6 + 3\pi/2}$ (B) $y = \frac{3p}{12 + 3\pi/2}$

(C) $y = \frac{3p}{12 + 5\pi/2}$ (D) $y = \frac{2p}{11 + 5\pi/2}$

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	B	C	B	D	B	A	C

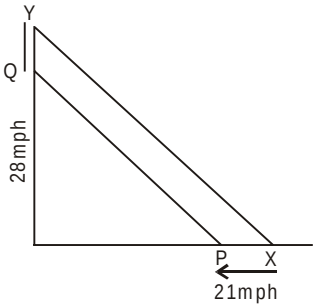
SOLUTIONS

Sol.1 (A)

Let two cars be at P and Q at time 't'

If the 1st car travels x ft,

then the 2nd car travels $\left(\frac{28}{21}x\right)$ ft in the same time.



$\therefore \quad PQ^2 = d^2$

$= (1500 - x)^2 + \left(2100 - \frac{4}{3}x\right)^2 = f(x)$

$\therefore \quad f'(x) = 2(1500 - x)(-1) + 2\left(2100 - \frac{4}{3}x\right)^2$

$= (-4/3)$

Let $f'(x) = 0$

$\Rightarrow \quad x = 1584$

$\therefore$ The least distance between two cars

$\Rightarrow \quad d = \sqrt{(1500 - 1548)^2 + (2100 - 2064)^2}$

$d = \sqrt{(48)^2 + (36)^2} = 60$ft

Sol.2 (B)

Replace $\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4}$ by $\cos \frac{x}{2}$

$\frac{dy}{dx} = -\frac{1}{2}(k^2 - 3k + 2)\sin \frac{x}{2} + (k - 1)$

$= -\frac{1}{2}(k - 1)(k - 2)\sin \frac{x}{2} + (k - 1)$

$= -\frac{(k-1)}{2}\left[(k - 2)\sin \frac{x}{2} - 2\right] \neq 0$

because the function does not possess critical points $k - 1 \neq 0$ and $\sin \frac{x}{2} = \frac{2}{k-2}$ does not possess any solution.

$\Rightarrow \quad k \neq 1$ and $\left|\frac{2}{k-2}\right| > 1$ as $\left|\sin \frac{x}{2}\right| \leq 1$

$\Rightarrow \quad k \neq 1$ and $|k - 2| < 2$

$\Rightarrow \quad k \neq 1$ and $-2 < (k - 2) < 2$ or $0 < k < 4$

Thus k lies in the interval (0, 4) but $k \neq 1$

i.e. $k \in (0, 1) \cup (1, 4)$.

Sol.3 (C)

Given that AB 60

Let the school be at a distance x from A (with 150 students), then

$D = 150x + 50(60 - x) = 100x + 3000$

D will be least and equal to 3000 if $x = 0$ i.e. school is built at A.

Sol.4 (B)

$\frac{dx}{dt} = a\left[-\sin t + \frac{1}{\tan(t/2)}\sec^2 \frac{t}{2}.\frac{1}{2}\right]$

$= a\left[-\sin t + \frac{1}{\sin t}\right]$

or $\frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}, \frac{dy}{dt} = a\cos t$

$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$

Length of tangent is $\frac{y}{y'}\sqrt{(1 + y'^2)}$

$\therefore \quad \frac{a\sin t}{\tan t}\sec t = a$

Sol.5 (D)

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \infty$

$\therefore \quad \frac{dx}{dt} = 2at = 0$

$\therefore \quad t = 0$

$\therefore \quad (x, y) = (0, 0)$

Sol. (6-B, 7-A, 8-C)

Let y be the length and x be the breadth of the rectangular portion Total perimeter of the window is $2x + 2y + (1/2)\pi y = p$(say). Let amount of light per square metre for the coloured glass be μ. If L is the total light transmitted then

$L = 3\mu \times$ Area of rectangular portion

$+ \mu \times$ Area of semi-circular portion

$$= 3\mu xy + \frac{1}{8}\mu \pi y^2$$

$$= \mu \left[\frac{3}{2}y\left(p - \left(2+\frac{\pi}{2}\right)y\right) + \frac{1}{8}\pi y^2\right]$$

$$\frac{dL}{dy} = \frac{\mu}{2}\left[3p - 6\left(2+\frac{\pi}{2}\right)y + \frac{\pi}{2}y\right]$$

$$\left.\frac{dL}{dy}\right|_{y=1} = \frac{\mu}{2}\left[3p - 12 - \frac{5\pi}{2}\right]$$

$$\frac{dL}{dy} = 0 \Rightarrow y = \frac{3p}{12 + 5\pi/2}$$

$$\frac{d^2L}{dy^2} = \frac{\mu}{2}\left(-12 - \frac{5\pi}{2}\right) < 0.$$

Therefore, for $y = 3p/[12 + (5\pi/2)]$, there will be maximum light transmission.

$$\frac{y}{x} = \frac{2.\dfrac{3p}{12+5\pi/2}}{p - \left(2+\dfrac{\pi}{2}\right)\dfrac{3p}{12+5\pi/2}}$$

$$= \frac{6}{(12+5\pi/2) - 3(2+\pi/2)} = \frac{6}{6+\pi}$$