Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-9(+3,–1). (Questions may have more than one option correct).

Q.1 The points of extrema of $f(x) = \int_0^x \frac{\sin t}{t} dt$ in the domain $x > 0$ are-

(A) $(2n + 1)\frac{\pi}{2}$; n = 1, 2, ...

(B) $(4n + 1)\frac{\pi}{2}$; n = 1, 2, ...

(C) $(2n + 1)\frac{\pi}{4}$; n = 1, 2, ...

(D) $n\pi$; n = 1, 2, ...

Q.2 If f(x) is strictly increasing function on an interval [a, b], then f^{-1} exist and it is also a-
(A) Strictly increasing
(B) Strictly decreasing
(C) Increasing (D) Decreasing

Q.3 If f(x) is continous on [a, b] such that $f(c) \leq 0$, $(f'(c) < 0)$ for each $c \in (a, b)$. Then f(x) is-
(A) Monotonically decreasing [a, b]
(B) Monotonically increasing on [a, b]
(C) Neither increasing nor decreasing on [a, b]
(D) None of these

Q.4 Let 'a' and 'b' be the lengths of the legs of a right triangle. If 'a' remains constant and 'b' increases continually, which of the following can describe how the ratio of the triangle's area to its perimeter changes {indicate all possible answers} ?
(A) Increases continually
(B) Decreases continually
(C) Remains constant
(D) First increases, and then decreases continually.

Q.5 N characters of information are held on magnetic tape, in batches of x characters each; the batch processing time is $\alpha + \beta x^2$ seconds; α and β are constants. The optimal value of x for fast processing is-

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

Q.6 The abscissa of the points of the curve $y = x^3$ in the interval [–2, 2], where the slope of the tangents can be obtained by mean value theorem for the interval [–2, 2] are-

(A) $\pm \frac{2}{\sqrt{3}}$ (B) $\pm \sqrt{3}$

(C) $\pm \frac{\sqrt{3}}{2}$ (D) 0

Q.7 Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$. Then f
(A) Is bounded
(B) Has a local maxima
(C) Has a local minima
(D) Is strictly increasing

Q.8 If the function $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all values of x, then-
(A) K < 1 (B) K > 1
(C) K < 2 (D) K > 2

Q.9 If $f(x) = |x - a|\,\phi(x)$, where $\phi(x)$ is a continuous, then-
(A) $f'(a) = \phi(a)$ (B) $f'(a-) = -\phi(a)$
(C) $f'(a+) = f'(a-)$ (D) None of these.

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	D	A	A	A	C	A	D	D	A,B

SOLUTIONS

Sol.1 (D)

$$f(x) = \int_0^x \frac{\sin t}{t}\,dt \Rightarrow f'(x) = \frac{\sin x}{x}$$

Put $f'(x) = 0 \Rightarrow \frac{\sin x}{x} = 0 \Rightarrow \sin x = 0$

$x = n\pi$, n = 1, 2, 3, ...

Sol.2 (A)

Sol.3 (A)

Sol.4 (A)

$$\frac{\text{area}}{\text{perimeter}} = \frac{\left(\frac{1}{2}\right)ab}{a+b+\sqrt{a^2+b^2}}$$

$$= \frac{\left(\frac{1}{2}\right)a}{\frac{a}{b}+1+\sqrt{\left(\frac{a}{b}\right)^2+1}}$$

As b increases, the denominator clearly decreases, so the ratio increases continually.

Sol.5 (C)

Here number of batches = $\frac{N}{x}$ and time per batch = $(\alpha + \beta x^2)$ second

$\therefore$ Total processing time

$$T = \left(\frac{N}{x}\right)(\alpha + \beta x^2) = N\left(\frac{\alpha}{x} + \beta x\right) \text{ second}$$

For fast processing T must be least

$$\therefore \quad \frac{dT}{dx} = N\left(-\frac{\alpha}{x^2} + \beta\right);\ \frac{d^2T}{dx^2} = \frac{2N\alpha}{x^3}$$

For maxima or minima of T, $\frac{dT}{dx} = 0$

$$\Rightarrow \quad x = \sqrt{\left(\frac{\alpha}{\beta}\right)}$$

For $x = \sqrt{\left(\frac{\alpha}{\beta}\right)}$, $\frac{d^2T}{dx^2}$ is +ve i.e., > 0

$\therefore$ T has minima for $x = \sqrt{\left(\frac{\alpha}{\beta}\right)}$

Sol.6 (A)

Given that equation of curve $y = x^3 = f(x)$

So $f(2) = 8$ and $f(-2) = -8$

Now $f'(x) = 3x^2 \Rightarrow f'(x) = \frac{f(2) - f(-2)}{2 - (-2)}$

$$\Rightarrow \quad \frac{8-(-8)}{4} = 3x^2;$$

$$\therefore \quad x = \pm\frac{2}{\sqrt{3}}$$

Sol.7 (D)

Given $f(x) = x^3 + bx^2 + cx + d$

$\therefore \quad f'(x) = 3x^2 + 2bx + c$

Now its discriminant $= 4(b^2 - 3c)$

$\Rightarrow \quad 4(b^2 - c) - 8c < 0$, as $b^2 < c$ and $c > 0$

Therefore, $f'(x) > 0$ for all $x \in R$

Hence f is strictly increasing.

Sol.8 (D)

Since $f(x) = \frac{K\sin x + 2\cos x}{\sin x + \cos x}$ is increasing for all x, therefore $f'(x) > 0$ for all x

$$\Rightarrow \quad \frac{K-2}{(\sin x + \cos x)^2} > 0 \text{ for all } x$$

$\Rightarrow \quad K - 2 > 0 \Rightarrow K > 2$

Sol.9 (A, B)

$$f'(a+) = \lim_{x\to a+}\frac{f(x)-f(a)}{x-a} = \lim_{x\to a+}\frac{|x-a|\phi(x)}{x-a}$$

$[\because f(a) = 0]$

$$= \lim_{x\to a+}\phi(x) = \phi(a)$$

[since for $x > a$, $|x - a| = x - a$]

and $f'(a-) = \lim_{x\to a-}\frac{f(x)-f(a)}{x-a}$

$$= \lim_{x\to a-}\frac{|x-a|\,\phi(x)}{x-a} = -\phi(a)$$