Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,−1). (Questions may have more then one option correct).

Q.1 The tangent at any point of the curve $x = a(t + \sin t \cos t)$, $y = a(1 + \sin t)^2$ makes angle

(A) $\left(\frac{\pi}{4}+\frac{1}{2}\right)$ with x axis (B) $\left(\frac{\pi}{2}+t\right)$ with x axis

(C) $\left(\frac{\pi}{2}+\frac{1}{2}\right)$ with y axis (D) $\left(\frac{\pi}{4}+2t\right)$ with y axis

Q.2 If tanA + tanB + tanC = 100 then there exist
(A) Exactly two non similar isosceles triangle
(B) Atleast two non similar isosceles triangle
(C) Exactly two similar isosceles triangle
(D) Atleast two similar isosceles triangle

Q.3 A cubic function f(x) vanishes at x = −2 and has relative minimum/maximum at x = −1 and $x = \frac{1}{3}$. If $\int_{-1}^{1} f(x)\,dx = \frac{14}{3}$, the cubic function f(x) is
(A) $x^3 - x + 2$ (B) $x^3 + x^2 - x + 2$
(C) $x^3 - x^2 + x - 2$ (D) $-x^3 - x^2 + x - 2$

Q.4 If the line ax + by + c = 0 is normal to the curve xy + 5 = 0, then
(A) a > 0, b > 0 (B) b > 0, a < 0
(C) a < 0, b < 0 (D) b < 0, a > 0

Q.5 The number of critical points of $f(x) = \max(\sin x, \cos x)$ for $x \in (0, 2\pi)$
(A) 2 (B) 5
(C) 3 (D) None

Q.6 The function $f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x - 3}$ is
(A) Max. at x = −3
(B) Min. at x = −3 and max. at x = 1
(C) Increasing in its domain
(D) None of these

Q.7 let $f(x) = \begin{cases} 1+\sin x, & x<0 \\ x^2 - x + 1, & x \ge 0 \end{cases}$. Then
(A) f has a local maximum at x = 0
(B) f has a local minimum at x = 0
(C) f is increasing every where
(D) f is decreasing everywhere

Q.8 If $f(x) = (\sin^2 x - 1)^n (2 + \cos^2 x)$, then $x = \pi/2$ is a point of
(A) Local maximum, if n is odd
(B) Local minimum, if n is odd
(C) Local maximum, if n is even
(D) Local minimum, if n is even

Q.9 Let N be any four digit number say, x_1, x_2, x_3, x_4. Then maximum value of $\frac{N}{x_1 + x_2 + x_3 + x_4}$ is equal to
(A) 1000 (B) $\frac{1111}{4}$
(C) 800 (D) None of these

Q.10 If S is the set such that $f(x) = 8x^2 - \ln|x|$ increases in S then S contains
(A) $\left(-\frac{1}{4}, 0\right)$ (B) (1, 2)
(C) $(-\infty, -1)$ (D) $(4, \infty)$

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 3) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	10	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭					
4	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	B	A,C	C	C	A	A,D	A	A,B,D

SOLUTIONS

Sol.1 (A)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$\because \frac{dx}{dt} = a\,(1 + \cos^2 t - \sin^2 t)$

$$\frac{dy}{dt} = 2a(1 + \sin t)\cos t$$

$$\frac{dy}{dx} = \frac{2(1+\sin t)\cos t}{2\cos^2 t} = \frac{1+\sin t}{\cos t}$$

$$= \frac{\left(\sin\frac{t}{2} + \cos\frac{t}{2}\right)^2}{\left(\cos^2\frac{t}{2} - \sin^2\frac{t}{2}\right)} = \frac{1+\tan\frac{t}{2}}{1-\tan\frac{t}{2}}$$

$= \tan \frac{\pi}{4} + \frac{t}{2}$ = slope of tangent = $\tan\theta$

If θ is the angle of tangent with x axis

$$\theta = \frac{\pi}{4} + \frac{t}{2} = \frac{1}{4}(\pi + 2t)$$

Sol.2 (A)

let A = B, then 2A + C = 180^0

and 2 tan A + tanc= 100

Now 2A + C $180^0 \Rightarrow$ tan2A = –tanc ...(1)

Also 2tanA + tanC = 100

$\Rightarrow$ 2tan A –100 = –tan C ...(2)

From (1) and (2)

$$2\tan A - 100 = \frac{2\tan A}{1-\tan^2 A}$$

let tanA = x

$$\Rightarrow \frac{2x}{1-x^2} = 2x - 100$$

$\Rightarrow x^3 - 50x^2 + 50 = 0$

Now let $f(x) = x^3 - 50x^2 + 50$

then $f'(x) = 3x^2 - 100x$

Thus f'(x) = 0 has roots 0, $\frac{100}{3}$

Also $f(0)\, f\left(\frac{100}{3}\right) < 0$

Thus, f(x) has exactly three distinct real roots. Therefore tanA and hence A has three distinct values. But one of them will be obtuse angle. Hence there exists exactly two non similar isosceles triangle.

Sol.3 (B)

$\because$ f(x) vanishes at x = –2

$\Rightarrow f(x) = (x + 2)(ax^2 + bx + c)$

$\Rightarrow f'(x) = (x + 2)(2ax + b) + (ax^2 + bx + c) = 0$

at x = –1 and x = $\frac{1}{3}$

–a + c = 0

and 15a + 24b + 9c = 0

$\Rightarrow$ 24a + 24b = 0

$\Rightarrow$ a + b = 0

$\Rightarrow$ c = a, b = –a

$\therefore f(x) = a(x + 2)(x^2 - x + 1)$

Again $\int_{-1}^{1} f(x)\,dx = \frac{14}{3}$ (given)

$$\Rightarrow \int_{-1}^{1} a(x + 2)(x^2 - x + 1)\,dx = \frac{14}{3}$$

$\Rightarrow$ a = 1

$\Rightarrow f(x) = x^3 + x^2 - x + 2$

Sol.4 (A, C)

$xy = -5 \Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} > 0$ [as xy = –5 < 0)

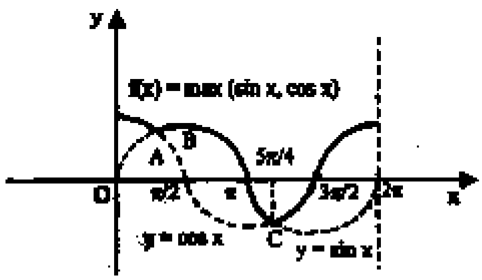
$\Rightarrow$ The slope of the normal is negative

$\Rightarrow \frac{-a}{b} < 0$

$\Rightarrow \frac{a}{b} > 0$

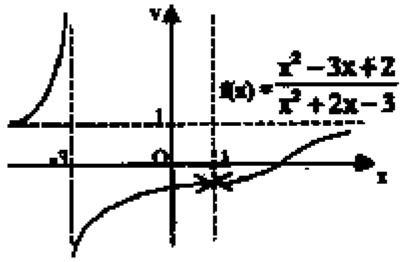
$\Rightarrow$ a > 0, b > 0 or a < 0, b < 0

Sol.5 (C)



Clearly A, B and C are the critical points.

Sol.6 (C)



$f(x) = \frac{x^2 - 3x + 2}{x^2 + 2x - 3} = \frac{(x-1)(x-2)}{(x-1)(x+3)} = \frac{x-2}{x+3}$,

$x \neq 1, -3$

$\frac{df}{dx} = \frac{(x+3)-(x-2)}{(x+3)^2} = \frac{5}{(x+3)^2} > 0 \;\; \forall x$

$\neq 1, -3$

Clearly f(x) is increasing in its domain.

Sol.7 (A)

f is continuous at '0' and f'(0–) > 0 and f'(0+) < 0. Thus f has a local maximum at '0'.

Sol.8 (A,D)

If x = a the point of local extremum of y = f(x),

then f(a – h). f(a + h) > 0

$\Rightarrow f(\pi/2 - h).\, f(\pi/2 + h) > 0$

$f(\pi/2 - h) = (-ve)^n$, $f(\pi/2 + h) = (-ve)^n$

$\Rightarrow f(\pi/2 - h).\, f(\pi/2 + h) = (-ve)^{2n} > 0$

$\Rightarrow$ n can be odd or even.

Sol.9 (A)

$$\frac{N}{x_1 + x_2 + x_3 + x_4}$$

$$= \frac{1000x_1 + 100x_2 + 10x_3 + x_4}{x_1 + x_2 + x_3 + x_4}$$

$$= 1000 - \frac{(900x_2 + 990x_3 + 999x_4)}{(x_1 + x_2 + x_3 + x_4)}$$

$\Rightarrow$ maximum value of $\frac{N}{x_1 + x_2 + x_3 + x_4}$

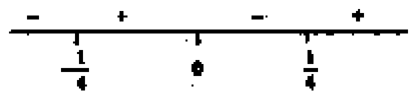
= 1000.

Sol.10 (A, B, D)

$$f'(x) = 16x - \frac{1}{x} = \frac{16}{x}\left(x^2 - \frac{1}{16}\right)$$

For an increasing function, f'(x) > 0

$$\Rightarrow \frac{1}{x}\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right) > 0$$



$$\Rightarrow x \in \left(-\frac{1}{4}, 0\right) \cup \left(-\frac{1}{4}, \infty\right)$$