Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-7(+3,−1),8(+6, 0) (Questions may have more then one option correct).

Q.1 If $27a + 9b + 3c + d = 0$, then the equation $4ax^3 + 3bx^2 + 2cx + d = 0$ has atleast one real root lying between

(A) 0 and 1 (B) 1 and 3

(C) 0 and 3 (D) None of these

Q.2 The value of c in Lagrange's theorem for the function $f(x) = \begin{cases} x\cos\frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$, $x \neq 0$ in the interval [−1, 1] is

(A) 0 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$

(D) Non-existent in the interval

Q.3 If f(x) and g(x) are differentiable function for $0 \leq x \leq 1$ such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$, $g(1) = 2$, then in the interval (0, 1)

(A) $f'(x) = 0 \ \forall \ x$

(B) $f'(x) = 2g'(x)$ for at least one x

(C) $f'(x) = 2g'(x)$ for at most one x

(D) None of these

Q.4 $f(x) = 2 + [\sin x]$, in $0 \leq x < \frac{\pi}{2}$

(A) Is continuous in $\left[0, \frac{\pi}{2}\right]$

(B) Has a maximum value 2

(C) Has a minimum value 1

(D) Is differentiable at $x = \frac{\pi}{2}$.

Q.5 Let $f(x) = (x-1)^2(x-2)^3 e^x$. Then f(x) has

(A) Local maximum at x = 1

(B) Point of inflection at x = 1

(C) Local minimum at x = 2

(D) Point of inflexion at x = 2.

Q.6 Tangents are drawn to y = cos x from the point P(0, 0). Points of contact of these tangents will always lie on ...

(A) $\frac{1}{x^2} = \frac{1}{y^2} + 1$ (B) $\frac{1}{x^2} = \frac{1}{y^2} - 1$

(C) $x^2 + y^2 = 1$ (D) $x^2 - y^2 = 1$.

Q.7 Total number of parallel tangents of $f_1(x) = x^2 - x + 1$ and .... $f_2(x) = x^3 - x^2 - 2x + 1$ is equal to

(A) 2 (B) 3

(C) 4 (D) None of these

Q.8 Match the list I with list II and select the correct answer using the codes given below the lists

List I functionf(x)	**List II** Interval of decrease
I $\cos\frac{\pi}{x}$	(A) $\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(0, \frac{1}{\sqrt{2}}\right)$
II $\frac{x}{\log_e x}$	(B) ϕ
III $\cot^{-1}x + x$	(C) $\left(\frac{1}{2K+2}, \frac{1}{2K+1}\right)$ K is a + ve integer
IV $x^2 - \log_e \lvert x \rvert$ $(x \neq 0)$	(D) $(0, 1) \cup (1, e)$ include '0'

(A) I-A, II-B, III-C, IV-D

(B) I-C, II-B, III-C, IV-A

(C) I-C, II-D, III-B, IV-A

(D) I-C, II-D, III-A, IV-B

MATHEMATICS IIT JEE (AUGUST 2nd WEEK CLASS TEST 4) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	C	D	B	B	A,D	B	D	C

SOLUTIONS

Sol.1 (C)

Let $f(x) = ax^4 + bx^3 + cx^2 + dx$ in $[0, 3]$

$f(0) = 0$

$f(3) = 81a + 27b + 9c + 3d$

$= 3(27a + 9b + 3c + d) = 0$

$\therefore$ $f(0) = f(3)$

Since f(x) is a polynomial

$\therefore$ It is continuous in [0, 3] and derivable in (0, 3)

Also, $f(0) = f(3)$

$\therefore$ By Rolle's Theorem, $\exists$ s at least one value between 0 and 3 such that $f'(x) = 0$

i.e., $4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one root between 0 and 3.

Sol.2 (D)

Here f(x) is continuous in [–1, 1], but f(x) is not diff. at $x = 0 \in (-1, 1)$

$\therefore$ Mean Value Theorem is not applicable.

$\therefore$ c cannot be found.

Sol.3 (B)

Let $h(x) = f(x) - 2g(x)$ $\forall x \in [0, 1]$

$\therefore$ $h'(x) = f'(x) - 2g'(x)$

$h(0) = f(0) - 2g(0) = 2 - 2(0) = 2$

$h(1) = f(1) - 2g(1) = 6 - 2(2) = 2$

$\therefore$ $h(0) = h(1)$

Since f(x) and g(x) are diff. in [0, 1]

$\therefore$ h(x) is diff. in [0, 1]

$\therefore$ it is cont. in [0, 1]

Also $h(0) = h(1)$

$\therefore$ by Rolle's Thm, $\exists$ s at least one $x \in (0, 1)$ such that $h'(x) = 0$

$\Rightarrow f'(x) - 2g'(x) = 0 \Rightarrow f'(x) = 2g'(x)$

Sol.4 (B)

$f(x) = 2 + [1 - 0] = 2$ = constant.

for $0 \le x \le \frac{\pi}{2}$

$\therefore$ f(x) is not continuous at $x = \frac{\pi}{2}$

Since $f\left(\frac{\pi}{2}\right)$ is not defined.

and therefore it is not diff. at $x = \frac{\pi}{2}$

Max. f(x) = 2, Min. value = 2

$[\because f(x) = 2\ \forall x \in \left[0, \frac{\pi}{2}\right)]$

Sol.5 (A, D)

$f'(x) = (x - 1)(x - 2)^2(x^2 + 2x - 5)e^x$.

Since f'(x) changes sign from + ve to – ve as we pass from value of $x < 1$ to $x > 1$

$\therefore$ f(x) has local maximum at $x = 1$

Again at $x = 2$, f'(x) does not change sign

$\therefore$ $x = 2$ is a point of inflexion.

Sol.6 (B)

$y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$

Let the point of contact be $[x_0, \cos x_0]$

Tangent at this point is

$y - \cos x_0 = -\sin x_0 (x - x_0)$

It point thro' (1, 0)

$\therefore$ $-\cos x_0 = -\sin x_0(-x_0)$

$\Rightarrow$ $x_0 = -\cot x_0$

Also, we have

$y_0 = \cos x_0$

$\therefore$ $\frac{1}{y_0^2} = \sec^2 x_0 = 1 + \tan^2 x_0$

$= 1 + \frac{1}{x_0^2}$

$\Rightarrow$ $\frac{1}{y_0^2} - \frac{1}{x_0^2} = 1$ $\therefore$ locus of (x_0, y_0) is

$\frac{1}{y^2} - \frac{1}{x^2} = 1 \Rightarrow \frac{1}{x^2} = \frac{1}{y^2} - 1$

Sol.7 (D)

$f'_1(x_1) = 2x_1 - 1$

$f'_2(x_2) = 3x_2^2 - 2x_2 - 2$

Let tangents drawn to the curves

$y = f_1(x)$

and $y = f_2(x)$ at $(x_1, f(x_1))$,

and $(x_2, f(x_2))$ be parallel.

$\therefore \quad 2x_1 - 1 = 3x_2^2 - 2x_2 - 2$

Which is possible for an infinite number of ordered pair (x_1, x_2)

Sol.8 (C)

I. $f'(x) < 0 \Rightarrow \sin\frac{\pi}{x} < 0$

$\Rightarrow (2K + 1)\pi < \frac{\pi}{x} < (2K + 2)\pi$

$\Rightarrow x \in \left(\frac{1}{2K+2}, \frac{1}{2K+1}\right) \quad \therefore \text{I} \leftrightarrow \text{(C)}$

II. $f'(x) < 0$ if $x < e \quad \therefore x \in (0, 1) \cup (1, e)$

II $\leftrightarrow$ (D)

III. $f'(x) \geq 0 \ \forall x \in R$

$\therefore$ $f'(x) < 0$ for no value of x

i.e. interval of decrease is ϕ. III $\leftrightarrow$ (B)

IV. $f'(x) < 0 \Rightarrow (\sqrt{2}x - 1)(\sqrt{2}x + 1) < 0$

$\Rightarrow x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(0, \frac{1}{\sqrt{2}}\right)$

$\therefore$ IV $\leftrightarrow$ (A)