Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Passage 1 :

If $f''(x) < 0$ (> 0) on an interval (a, b) then the curve $y = f(x)$ on this interval is convex (concave), i.e. it is situated below (above) any of its tangent lines. If $f''(x_0) = 0$ or does not exist but $f'(x_0)$ does exist and the second derivative changes sign when passing through the point x_0 then the point $(x_0, f(x_0))$ is the point of inflection of the curve $y = f(x)$.

Q.1 If $y = x^4 + x^3 - 18x^2 + 24x - 12$ then
(A) $(-2, -24)$ is a point of inflection
(B) $(-2, 3/2)$ is a point of inflection
(C) $\left(\frac{3}{2}, -8\frac{1}{16}\right)$ is a point of inflection
(D) y is convex on $\left(\frac{3}{2}, \infty\right)$

Q.2 If $y = x \sin(\log x)$ then
(A) y has only two points of inflection
(B) y has only four points of inflection
(C) $(\pi/4)$ is the only point of inflection
(D) y has infinite number of points of inflection

Q.3 $y = x^4 + ax^3 + \frac{3}{2}x^2 + 1$ is concave along the entire number scale then
(A) $|a| \geq 1$ (B) $|a| \leq 1$
(C) $|a| \leq 2$ (D) $|a| > 2$

Passage 2 (Q.No. 4-5):

Let f(x) be a continuous, differentiable and bijective function. If the tangent to $y = f(x)$ at $x = a$, is also the normal to $y = f(x)$ at $x = b$. Then

Q.4 We must have atleast one value c belonging to (for which $f'(c) = 0$)
(A) $\left(\frac{a}{2}, b\right)$ (B) $\left(a, \frac{b}{2}\right)$ (C) (0, a) (D) None

Q.5 At the point $x = c$, we will have a point of;
(A) local maxima
(B) local minima
(C) nothing can be said
(D) None of these

Q.6 For the parabola $y^2 = 16x$, the ratio of the length of the subtangent to the abscissa is-
(A) 2 : 1 (B) 1 : 1 (C) x : y (D) $x^2 : y$

Q.7 An object is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$. As it passes through the point $\left(1/2, \sqrt{3}/2\right)$, its y-coordinate is decreasing at the rate of 3 units per second. The rate at which the x-coordinate changes at this point is (in units per second)
(A) 2 (B) $3\sqrt{3}$ (C) $\sqrt{3}$ (D) $2\sqrt{3}$

Q.8 The values of x for which the tangents to the curves $y = x \cos x$, $y = (\sin x)/x$ are parallel to the axis of x are roots of (respectively)
(A) $\sin x = x$, $\tan x = x$
(B) $\cot x = x$, $\sec x = x$
(C) $\cot x = x$, $\tan x = x$
(D) $\tan x = x$, $\cot x = x$

Q.9 If $y = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$, then y
(A) Decreases on $(-\infty, \infty)$
(B) Decreases on $[0, \infty)$
(C) Neither decreases nor increases on $[0, \infty)$
(D) Increases on $(-\infty, \infty)$

Q.10 If the tangent at (1, 1) on $y^2 = x(2 - x)^2$ meets the curve again at P, then P is-
(A) (4, 4) (B) (−1, 2)
(C) (9/4, 3/8) (D) None of these

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	C	D	D	A	B	C	D	C

SOLUTIONS

Sol.1 (C)

$y' = 4x^3 + 3x^2 - 36x + 24$

$y'' = 12x^2 + 6x - 36 = 12\left(x^2 + \frac{x}{2} - 3\right)$

whence $y'' = 0$ at $x_1 = -2$, $x_2 = \frac{3}{2}$

Also $y'' > 0$ on $(-\infty, -2)$ and $\left(\frac{3}{2}, \infty\right)$:

$y'' < 0$ on $\left(-2, \frac{3}{2}\right)$

so the points $(-2, -124)$ $\left(\frac{3}{2}, -8\frac{1}{16}\right)$ are points of inflection and y is concave on $\left(\frac{3}{2}, \infty\right)$.

Sol.2 (D)

$y'' = 12x^2 + 6ax + 3$. The curve will be concave along the entire number scale of $y'' \geq 0$ for all values of x i.e. when

$4x^2 + 2ax + 1 \geq 0$ for all x

For this it is necessary and sufficient that the inequality $4a^2 - 16 \leq 0$ be fulfilled, whence $|a| \leq 2$.

Sol.3 (C)

$y' = \sin(\log x) + \cos(\log x)$ and

$y'' = \frac{1}{x}[\cos(\log x) - \sin(\log x)]$

$= \frac{\sqrt{2}}{x}\sin\left(\frac{\pi}{4} - \log x\right)$

$y'' = 0$ at $x_k = e^{\pi/4 + k\pi}$, $k = 0, \pm1, \pm2, ...$

The function $\sin\left(\frac{\pi}{4} - \log x\right)$ and so y", changes sign when passing through each point x_k. Consequently, the points x_k are the abscissas of points of inflection.

Sol. (4 - 5)

4 - (D), 5 - (D)

Since the same line is tangent at one point x = a and normal at other point x = b.

$\Rightarrow$ Tangent at x = b will be perpendicular to tangent at x = a.

$\Rightarrow$ Slope of tangent goes from positive to negative or negative to positive.

$\Rightarrow$ It takes the value zero some where.

$\therefore$ There will be a point 'c' $\in$ (a, b), where $f'(c) = 0$.

Since f(x) is bijective, therefore x = c is neither a point of maximum nor of minimum.

Sol.6 (A)

Differentiating, $2y\frac{dy}{dx} = 16$ so $\frac{dy}{dx} = \frac{8}{y}$.

Thus the length of the subtangent is $y.\frac{dx}{dy}$

$= \frac{y^2}{8} = \frac{16x}{8} = 2x$. Hence length of the subtangent : abscissa = 2x : x = 2 : 1.

Sol.7 (B)

We find $\frac{dx}{dt}$ when $x = \frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$ given that $\frac{dy}{dt} = -3$ units and $x^2 + y^2 = 1$.

Differentiating $x^2 + y^2 = 1$, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Putting x = 1/2, $y = \frac{\sqrt{3}}{2}$ and dy/dt = – 3, we have

$$\frac{1}{2}\frac{dx}{dt} + \frac{\sqrt{3}}{2}(-3) = 0$$

$$\Rightarrow \frac{dx}{dt} = 3\sqrt{3}$$

Sol.8 (C)

Let y = f(x) = x cos x

and y = g(x) = (sin x)/x.

Now f'(x) = – x sin x + cos x

and g'(x) = (x cos x – sin x)/x^2.

Since the tangents are parallel to x-axis so f'(x) = 0 and g'(x) = 0, which is turn give cot x = x and tan x = x respectively.

Sol.9 (D)

$$y'(x) = 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}-x}\frac{d}{dx}\left(\sqrt{1+x^2}-x\right)$$

$$= 2 - \frac{1}{1+x^2} + \frac{1}{\sqrt{1+x^2}-x} \times \left(\frac{x}{\sqrt{1+x^2}}-1\right)$$

$$= 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} \geq 0$$

since $1/(1 + x^2)$ and $\frac{1}{\sqrt{1+x^2}}$ are less than or equal to 1 for all x. So f(x) increases on (–∞, ∞).

Sol.10 (C)

$$2y\frac{dy}{dx} = (2-x)^2 - 2x(2-x) = 3x^2 - 8x + 4.$$

So $\left.\frac{dy}{dx}\right|_{(1,1)} = -1/2$. An equation of tangent at (1, 1) is $Y - 1 = \left(-\frac{1}{2}\right)(X-1)$

i.e. Y = (–1/2)x + 3/2

The intersection of this line with the given curve is given by $(-x/2 + 3/2)^2 = x(2-x)^2$

$\Rightarrow$ $x^2 - 6x + 9 = 16x + 4x^3 - 16x^2$.

$\Rightarrow$ (x – 1) (4x – 9) (x + 1) = 0.

Thus x = 1, 9/4, – 1. But x = – 1 cannot lie on the given curve so required point is $\left(\frac{9}{4}, \frac{3}{8}\right)$