Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,–1). (All questions have only one option correct).

Q.1 If the length of subnormal is equal to length of subtangent at any point (3, 4) on the curve y = f(x) and the tangent at (3, 4) to y = f(x) meets the coordinate axes at A and B, then maximum area of the ΔOAB where O is origin is :

(A) $\frac{45}{2}$ (B) $\frac{49}{2}$ (C) $\frac{25}{2}$ (D) $\frac{81}{2}$

Q.2 If f"(x) < 0 for all x ∈ (a, b), then f'(x) = 0
(A) exactly once in (a, b)
(B) at most once in (a, b)
(C) atleast once in (a, b)
(D) None of these

Q.3 If a, b, c be non-zero real numbers such that

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\,dx = 0$$

Then the equation $ax^2 + bx + c = 0$ will have
(A) one root between 0 and 1 other root between 1 and 2
(B) both roots between 0 and 1
(C) both roots between 1 and 2
(D) None of these

Q.4 If $0 < a < b < \frac{\pi}{2}$ and $f(a, b) = \frac{\tan b - \tan a}{b - a}$, then-
(A) f(a, b) ≥ 2 (B) f(a, b) > 1
(C) f(a, b) ≤ 1 (D) None of these

Q.5 If f(x) is a polynomial of degree 5 with real coefficients such that f(|x|) = 0 has 8 real roots then f(|x|) = 0 has ;
(A) 4 real roots (B) 5 real roots
(C) 3 real roots (D) Nothing can be said

Q.6 Let f(x) satisfies the requirement of lagrange's mean value theorem in [0, 2]. If f(0) = 0 and |f'(x)| ≤ 1/2 for all x ∈ [0, 2], then-
(A) f(x) ≤ 2
(B) |f(x)| ≤ 2x
(C) |f(x)| ≤ 1
(D) f(x) = 3, for at least one x ∈ [0, 2]

PASSAGE :

A manufacturing plant has a capacity of 25 articles per week. Experience has shown that x articles per week can be sold at a price of Rs p each where p = 110 – 2x and the cost of producing x articles is $600 + 10x + x^2$ Rs.

Q.7 The profit function represents a
(A) straight line
(B) a parabola with vertex at $\left(20, \frac{700}{3}\right)$
(C) a parabola with vertex at $\left(\frac{50}{3}, \frac{700}{3}\right)$ and focus $\left(\frac{50}{3}, \frac{2199}{3}\right)$
(D) a parabola with vertex at $\left(\frac{50}{3}, \frac{700}{3}\right)$ and focus $\left(\frac{50}{3}, \frac{2099}{12}\right)$

Q.8 Integral value of x for which the profit function is maximum
(A) 24 (B) 22 (C) 17 (D) 16

Q.9 If P is the profit function then
(A) P increases on [8, 25]
(B) P decreases on [8, 25]
(C) P decreases on [16(2/3), 25]
(D) P increases on [8, 22]

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	B	A	A	B	B	C	D	C	C

SOLUTIONS

Sol.1 (B)

Length of subnormal = length of subtangent

$\Rightarrow \quad \frac{dy}{dx} = \pm 1$

If $\frac{dy}{dx} = 1$, equation of tangent is

$y - 4 = x - 3$

$\Rightarrow \quad y - x = 1$

Area of $\Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ ...(i)

If $\frac{dy}{dx} = -1$, equation of tangent is

$y - 4 = - x + 3$

$\Rightarrow \quad x + y = 7$

Area of $\Delta OAB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}$ ...(ii)

$\therefore$ Maximum area $= \frac{49}{2}$

Sol.2 (A)

If possible, let x_1, x_2 be two distinct points in (a, b) such that $f'(x_1) = f'(x_2) = 0$.

Then, by Rolle's theorem there exists a point $c \in (a, b)$ such that $f''(c) = 0$. This contradicts the given condition that $f''(x) < 0$. For all $x \in (a, b)$.

Hence, our supposition is wrong. Consequently, there can be at most one point in (a, b) at which f'(x) is zero.

Sol.3 (A)

Consider the function $\phi(x)$ given by

$$\phi(x) = \int_0^x (1+\cos^8 t)(at^2+bt+c)\,dt$$

$\Rightarrow \quad \phi'(x) = (1 + \cos^8 x)(ax^2 + bx + c)$ ...(i)

We observe that

$\phi(0) = 0$

$\phi(1) = \int_0^1 (1+\cos^8 t)(at^2+bt+c)\,dt$ (given)

and, $\phi(2) = \int_0^2 (1+\cos^8 t)(at^2+bt+c)\,dt$ (given)

$\therefore$ 0, 1 and 2 are the roots of $\phi(x)$

By Rolle's Theorem $\phi'(x) = 0$ will have atleast one real root between 0 and 1 and at least one real root between 1 and 2.

Sol.4 (B)

Consider the function f(x) = tan x, defined on [a, b] such that $a, b \in \left(0, \frac{\pi}{2}\right)$

Applying lagrange's mean value theorem, we have

$$f'(c) = \frac{f(b)-f(a)}{b-a} \text{ for some } c \in (a, b)$$

$\Rightarrow \quad \sec^2 c = \frac{\tan b - \tan a}{b-a}$

$\Rightarrow \quad f(a, b) = \sec^2 c$

$\Rightarrow \quad f(a, b) > 1$

$[\because \sec^2 c > 1 \text{ as } c \in (0, \pi/2)]$

Sol.5 (B)

Given that f(|x|) = 0 has 8 real roots

$\Rightarrow$ f(x) = 0 has 4 positive roots.

Since f(x) is a polynomial of degree 5, f(x) cannot have even number of real roots.

$\Rightarrow$ f(x) has all the five roots real in which four positive and one root is negative.

Sol.6 (C)

Let $x \in (0, 2)$. Since f(x) satisfies the requirement of lagrange's mean value theorem in [0, 2]. So, it also satisfies in [0, x]. Consequently, there exists $c \in (0, x)$ such that

$$f'(c) = \frac{f(x)-f(0)}{x-0}$$

$\Rightarrow \quad f'(c) = \frac{f(x)}{x}$

$\Rightarrow \quad \left|\frac{f(x)}{x}\right| = |f'(c)| \le 1/2$

$\{\because |f'(x)| \le 1/2\}$

$\Rightarrow \quad |f(x)| \le \frac{x}{2}$

$\Rightarrow \quad |f(x)| \le 1$

$\{\because x \in [0, 2], \therefore |x| \le 2\}$

Sol. (7 - 9)

7 - (D), 8 - (C), 9 - (C)

P = Profit = Revenue − Cost

$= xp - (600 + 10x + x^2)$

$= x(110 - 2x) - (600 + 10x + x^2)$

$= 110x - 600 - 3x^2$

$= -3\,[(x - 50/3)^2 - 700/9]$

So $P - \frac{700}{9} = -3\left(x - \frac{50}{3}\right)^2$. This represents a parabola with vertex at $\left(\frac{50}{3}, \frac{700}{3}\right)$ and focus $\left(\frac{50}{3}, \frac{2099}{12}\right)$. Now P is negative if x < 8 or greater 25. Thus it is enough to consider

$P(x) = 100x - 600 - 3x^2,$

$8 \le x \le 25$

$P'(x) = 100 - 6x = 0$

$\Rightarrow$ $x = 16\frac{2}{3}$. Since $f'(x) > 0$ on $\left(8, 16\frac{2}{3}\right)$, f increases on $\left(8, 16\frac{2}{3}\right)$ and $P'(x) < 0$ on $\left(16\frac{2}{3}, 25\right)$, f decreases on $\left(16\frac{2}{3}, 25\right)$. The largest value of P corresponding to an integer value of x will therefore occur at x = 16 or x = 17. Direct calculation of P(16) and P(17) shows the choice x = 17 is correct.