Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(24) + g^2(24)$ is-

(A) 32 (B) 24 (C) 64 (D) 48

Q.2 The solution set of $f'(x) > g'(x)$ where $f(x) = (1/2)\, 5^{2x+1}$ and $g(x) = 5^x + 4x \log 5$ is-

(A) $(1, \infty)$ (B) $(0, 1)$
(C) $[0, \infty)$ (D) $(0, \infty)$

Q.3 The set onto which the derivative of the function $f(x) = x(\log x - 1)$ maps the ray $[1, \infty)$ is

(A) $[1, \infty)$ (B) $(0, \infty)$
(C) $[0, \infty)$ (D) None of these

Q.4 The least value of n so that $y_n = y_{n+1}$ where $y = x^2 + e^x$ is-

(A) 4 (B) 3 (C) 5 (D) 2

Q.5 The length of the subtangent to the ellipse $x = a \cos t,\ y = b \sin t$ at $t = \frac{\pi}{4}$ is-

(A) a (B) (C) $\frac{b}{\sqrt{2}}$ (D) $\frac{a}{\sqrt{2}}$

Q.6 If $f(x) = e^x - e^{-x} - 2 \sin x - \frac{2}{3}x^3$, then the least value of n for which $\frac{d^n}{dx^n} f(x)$ at $x = 0$ is non-zero is-

(A) 2 (B) 1
(C) 7 (D) Either 1 or 2

Q.7 If $y = (1 + x)(1 + x^2)(1 + x^4) \ldots\ldots (1 + x^{2n})$, then at $x = 0$, $\frac{dy}{dx} =$

(A) –1 (B) 0
(C) 1 (D) None of these

Q.8 If $f(0) = 0$, $f'(0) = 2$ then the derivative of $y = f(f(f(f(x))))$ at $x = 0$ is-

(A) 2 (B) 8 (C) 16 (D) 4

Q.9 If the normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3)$ is equal to-

(A) – 1 (B) – 3/4
(C) 4/3 (D) 1

Q.10 The curve $y = ax^3 + bx^2 + cx + 8$ touches x-axis at P(– 2, 0) and cuts the y-axis at a point Q where its gradient is 3. The values of a, b, c are respectively-

(A) – 1/2, – 3/4, 3 (B) 3, –1/2, –4
(C) –1/2, –7/4, 2 (D) None of these

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 3) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	D	C	B	D	C	C	C	D	D

SOLUTIONS

Sol.1 (A)

$\frac{d}{dx}[f^2(x) + g^2(x)]$

$= 2f(x)\,f'(x) + 2g(x)\,g'(x)$

$= 2f(x)\,g(x) - 2g(x)\,f(x)$

$= 0$

$\Rightarrow f^2(x) + g^2(x) = \text{constant}$

$\therefore f^2(24) + g^2(24)$

$= f^2(2) + g^2(2)$

$= (4)^2 + (f'(2))^2$

$= (4)^2 + (4)^2 = 32$

Sol.2 (D)

$f'(x) = (1/2)\,5^{2x+1}(\log 5)(2) = \log 5 . (5^{2x+1})$

also $g'(x) = 5^x \log 5 + 4 \log 5$.

So $\{x : f'(x) > g'(x)\}$

$= \{x : \log 5 . 5^{2x+1} > \log 5\, 5^x + 4 \log 5\}$

$= \{x : 5^{2x+1} > 5^x + 4\}$

$= \{t = 5^x : 5t^2 - t - 4 > 0\}$

$= \{t = 5^x : (5t + 4)(t - 1) > 0\}$

$= \{t = 5^x : t > 1 \text{ or } t < -4/5\}$

$= \{t = 5^x : t > 1\} = (0, \infty)$.

Sol.3 (C)

$f'(x) = \log x - 1 + x\left(\frac{1}{x}\right) = \log x$

Since log x is an increasing function so f' maps $[1, \infty)$ onto $[0, \infty)$.

Sol.4 (B)

$y' = 2x + e^x$, $y'' = 2 + e^x$, $y''' = e^x$, $y^{iv} = e^x$.

Thus n = 3.

Sol.5 (D)

$\frac{dx}{dt} = -a \sin t$ and $\frac{dy}{dt} = b \cos t$,

therefore $\left.\frac{dy}{dt}\right|_{x=\pi/4} = -\frac{b}{a}\cot\left(\frac{\pi}{4}\right) = -\frac{b}{a}$.

The length of the subtangent

$= \left|y_0 \frac{dx}{dy}\right|_{x=\pi/4} = \left|b \sin\frac{\pi}{4} \times -\frac{a}{b}\right| = \frac{a}{\sqrt{2}}$.

Sol.6 (C)

$f(x) = 2(\sin hx - \sin x) - \frac{2}{3}x^3$

$= 2\left[x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ...\right]$

$- 2\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} ....\right] - \frac{2}{3}x^3$

$\therefore f(x) = 4\frac{x^7}{7!}$ .....

Now $\frac{d^7}{dx^7} f(x) = \frac{4}{7!} 7! = 4$, i.e., constant.

Hence at x = 0, it will be non-zero. Hence n = 7.

Sol.7 (C)

$y = \frac{(1-x)(1+x)(1+x^2)(1+x^4)......(1+x^{2n})}{(1-x)}$

$= \frac{1}{(1-x)}(1-x^2)(1+x^2)(1+x^4) - (1+x^{2n})$

$y = \frac{1}{1-x}(1 - x^{2n+1})$

$\Rightarrow \frac{dy}{dx} = \frac{-(1-x)(2n+1)(x^{2n}) + (1-x^{2n+1})}{(1-x)^2}$

$= \frac{-(2n+1)[x^{2n} - x^{2n+1}] + 1 - x^{2n+1}}{(1-x)^2}$

Now put x = 0

$\therefore \frac{dy}{dx} = \frac{1}{1} = 1$

Sol.8 (C)

$y'(x) = f'(f(f(f(x)))\, f'(f(f(x)))\, f'(f(x))\, f'(x)$

so $y'(0) = f'(f(f(f(0)))\, f'(f(f(0)))\, f'(f(0))\, f'(0)$

$= f'(f(f(0)))\, f'(f(0))\, f'(0)\, f'(0)$

$= (f'(0))^4 = 2^4 = 16$

Sol.9 (D)

According to the given condition

$$-\frac{1}{f'(3)} = \tan\frac{3\pi}{4} = -1$$

$\Rightarrow$ $f'(3) = 1$

Sol.10 (D)

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

Since the curve touches x-axis at (–2, 0) so

$$\left.\frac{dy}{dx}\right|_{(-2,\,0)} = 0$$

$\Rightarrow$ $12a - 4b + c = 0$ ...... (1)

The curve cut the y-axis at (0, 8)

so $\left.\frac{dy}{dx}\right|_{(0,\,8)} = 3 \Rightarrow c = 3$

Also the curve passes through (–2, 0) so

$0 = -8a + 4b - 2c + 8$

$\Rightarrow$ $-8a + 4b - 2 = 0$ .... (2)

Solving (1) and (2) $a = -\frac{1}{4}$, $b = 0$.