*Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (Questions may have more than one option correct).*

**Q.1** Let $f(x) = x^n$, n being a non-negative integer. The value of n for which the equality $f'(x + y) = f'(x) + f'(y)$ is valid for all x, y > 0 is-
(A) 0 (B) 1
(C) 2 (D) None of these

**Q.2** If $f(x) = |x - 2|$ and $g(x) = f(f(x))$, then for x > 20, $g'(x)$ equals-
(A) –1 (B) 1
(C) 0 (D) None of these

**Q.3** If $x^2 + y^2 = a^2$ and $k = 1/a$, then k is equal to-
(A) $\frac{y''}{\sqrt{1+y'}}$ (B) $\frac{|y''|}{\sqrt{(1+y'^2)^3}}$
(C) $\frac{2y''}{\sqrt{1+y'^2}}$ (D) $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

**Q.4** The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point x = 0 is-
(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{-1}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{3}}$

**Q.5** The tangent to the curve $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ at the points $\theta = (2k+1)\pi$, $k \in z$ are parallel to-
(A) y = x (B) y = – x
(C) y = 0 (D) x = 0

**Q.6** If $t = \frac{v^2}{2}$, then $\left(-\frac{df}{dt}\right)$ is equal to,
(where f is acceleration)
(A) $f^2$ (B) $f^3$ (C) $-f^3$ (D) $-f^2$

**Q.7** The equation of motion of a stone, thrown vertically upwards is $s = ut - 6.3t^2$, where the units of s and t are cm and sec. If the stone reaches at maximum height in 3 sec, then u =
(A) 18.9 cm/sec (B) 12.6 cm/sec
(C) 37.8 cm/sec (D) None of these

**Q.8** At what point on the curve $x^3 - 8a^2y = 0$, the slope of the normal is $\frac{-2}{3}$
(A) (a, a) (B) (2a, – a)
(C) (2a, a) (D) None of these

**Q.9** The curves $\frac{x^2}{a^2+\lambda_1} + \frac{y^2}{b^2+\lambda_1} = 1$ and $\frac{x^2}{a^2+\lambda_2} + \frac{y^2}{b^2+\lambda_2} = 1$
intersect at an angle-
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) None

**Q.10** The function $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3$, $0 \le x \le \pi$ is-
(A) Increasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(B) Increasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$
(C) Decreasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(D) None of these

## MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 4) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .................................................................................... Roll No. : ................................

| | A | B | C | D | | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ○ | ○ | ○ | ○ | 4 | ○ | ○ | ○ | ○ | 7 | ○ | ○ | ○ | ○ |
| 2 | ○ | ○ | ○ | ○ | 5 | ○ | ○ | ○ | ○ | 8 | ○ | ○ | ○ | ○ |
| 3 | ○ | ○ | ○ | ○ | 6 | ○ | ○ | ○ | ○ | 9 | ○ | ○ | ○ | ○ |
| | | | | | | | | | | 10 | ○ | ○ | ○ | ○ |

## ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | B,C | B | B | A | C | B | C | C | C | A |

# SOLUTIONS

**Sol.1 (B,C)**

We have $f(x) = x^n$

$\Rightarrow f(x + y) = (x + y)^n$

$\Rightarrow f'(x + y) = n(x + y)^{n-1}$

Also, $f'(x) = n x^{n-1}$ and $f'(y) = ny^{n-1}$

$\therefore f'(x + y) = f'(x) + f'(y)$

$\Rightarrow n(x + y)^{n-1} = n.x^{n-1} + n.y^{n-1}$

$\Rightarrow (x + y)^{n-1} = x^{n-1} + y^{n-1}$ ........(1)

For $n - 1 > 1$, we find that LHS of (1) is greater than the RHS. So, we must have $n - 1 \le 1$ i.e. $n - 1 = 0$ or $n - 1 = 1$. Hence, $n = 1$ or $n = 2$.

**Sol.2 (B)**

For $x > 20$, we have

$f(x) = |x - 2| = x - 2$

and, $g(x) = f(f(x)) = f(x - 2)$

$= x - 2 - 2 = x - 4$

$\therefore g'(x) = 1.$

**Sol.3 (B)**

$x^2 + y^2 = a^2 \Rightarrow 2x + 2yy_1 = 0 \Rightarrow y_1 = -x/y$

Now, $yy_1 + x = 0$

$\Rightarrow yy_2 + y_1^2 + 1 = 0$

$$\Rightarrow y = -\frac{1+y_1^2}{y_2} \qquad \text{........ (1)}$$

$$\therefore k = \frac{1}{a} = \left|\frac{1}{\sqrt{x^2+y^2}}\right| = \left\{\frac{1}{y\sqrt{1+\frac{x^2}{y^2}}}\right\}$$

$$= \left\{\frac{1}{y\sqrt{1+y_1^2}}\right\} \qquad [\because \quad y_1 = -x/y]$$

$$= \left\{\frac{y''}{(1+y_1^2)\sqrt{1+y_1^2}}\right\} = \frac{|y''|}{(1+y_1^2)^{3/2}}$$

**Sol.4 (A)**

Putting $x = 0$ in the given curve, we obtain $y = 1$. So, the given point is $(0, 1)$.

Now, $y = e^{2x} + x^2$

$$\Rightarrow \frac{dy}{dx} = 2e^{2x} + 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = 2.$$

The equation of the tangent at (0, 1) is

$y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0$ ...(1)

Required distance = length of the $\perp$ from

(0, 0) on (1) $= \frac{1}{\sqrt{5}}$

**Sol.5 (C)**

We have, $\frac{dy}{dx} = \frac{-\sin\theta}{1-\cos\theta}$

Clearly, $\frac{dy}{dx} = 0$ for $\theta = (2k + 1)\pi$.

So, the tangent is parallel to x-axis i.e. $y = 0$

**Sol.6 (B)**

$$t = \frac{v^2}{2} \Rightarrow v^2 = 2t \Rightarrow 2v\frac{dv}{dt} = 2$$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{v} = f$$

$$\Rightarrow \frac{df}{dt} = -\frac{1}{v^2}\frac{dv}{dt} = -\frac{1}{v^2} \times \frac{1}{v}$$

$$\Rightarrow -\frac{df}{dt} = \frac{1}{v^3} = f^3.$$

**Sol.7 (C)**

Velocity of particle at the maximum height

$\frac{ds}{dt} = 0$

$\Rightarrow u - 12.6t = 0 \Rightarrow u = 12.6t = 37.8$ cm/s.

**Sol.8 (B)**

$$x^3 - 8a^2y = 0 \Rightarrow 3x^2 - 8a^2 . \frac{dy}{dx} = 0$$

$\Rightarrow \quad 3x^2 = 8a^2 \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{3x^2}{8a^2}$

$\therefore$ Slope of the normal

$$= -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{1}{\frac{3x^2}{8a^2}} = -\frac{8a^2}{3x^2}$$

Given $\frac{-8a^2}{3x^2} = \frac{-2}{3}$ $\quad \therefore$ (x, y) = (2a, a).

**Sol.9 (C)**

Since $\frac{1}{\frac{1}{a^2+\lambda_1}} - \frac{1}{\frac{1}{a^2+\lambda_2}}$

and $\frac{1}{\frac{1}{b^2+\lambda_1}} - \frac{1}{\frac{1}{b^2+\lambda_2}}$

equals to $(a^2 + \lambda_1) - (a^2 + \lambda_2)$

and $(b^2 + \lambda_1) - (b^2 + \lambda_2)$

and $\lambda_1 - \lambda_2 = \lambda_1 - \lambda_2$

$\therefore$ curves cut orthogonally.

**Sol.10 (A)**

$f'(x) = 12 \cos^3 x \cdot (-\sin x) + 30 \cos^2 x$

$(-\sin x) + 12 \cos x (-\sin x)$

$= -6 \sin x \cos x [\cos x + 2] [2 \cos x + 1]$

$f'(x) = 0 \Rightarrow x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

Clearly, $f'(x) > 0$ for $\frac{\pi}{2} < x < \frac{2\pi}{3}$

and $f'(x) < 0$ for $0 < x < \frac{\pi}{2}$

or $\frac{2\pi}{3} < x < \pi$

Thus f(x) is increasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and

decreasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$.