Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 On the curve $x^3 = 12y$ the abscissa changes at a faster rate then the ordinate, then x belongs to interval

(A) (–2, 2) (B) (–1, 1)
(C) (0, 2) (D) None of these

Q.2 The graph of the function y = f(x) has a unique tangent at the point (a, 0) through which it passes. Then $\lim_{x\to a} \frac{\log_e\{1+6f(x)\}}{3f(x)}$ equals

(A) 1 (B) 0
(C) 2 (D) None of these

Q.3 If m is the slope of tangent to the curve $e^y = 1 + z^2$, then

(A) |m| > 1 (B) |m| < 1
(C) |m| ≤ 1 (D) None of these

Q.4 The set of points where $f(x) = (x-1)^2 (x + |x-1|)$ is thrice differentiable, is

(A) R (B) R – {0}
(C) R – {1} (D) R – {0, 1}

Q.5 The two curves $y = x^3 + ax - 1$ and $y = 6x^2 + b$ touch each other at a point having abscissca 1 when

(A) a = 3, b = –3 (B) a = 9, b = 3
(C) a = 0, b = –6 (D) a = 3, b = 9

Q.6 The equation of normal to the curve $y = \int_0^x \sin^3 t\,dt$ at $x = \pi/2$ is

(A) $x - y = \pi/2 - 2/3$ (B) $x + y = \pi/2 + 2/3$
(C) $x - y = \pi/6$ (D) $x + y = 5\pi/6$

Q.7 Let $f(x) = x^{x^x} + (x^x)^x$. Then f'(1) is equal to

(A) 1 (B) 2
(C) 3 (D) 4

Q.8 If a tangent to the curve xy + ax + by = 0 at (1, 1) is inclined at an angle $\tan^{-1} 2$ with x axis, then

(A) a = 1, b = 1 (B) a = 1, b = –2
(C) a = –1, b = 2 (D) a = –1, b = –2

Q.9 If $f(x) = \sqrt{x^2 - 2x + 1}$, then-

(A) f'(x) = 1 for all x
(B) f'(x) = – 1 for all x ≤ 1
(C) f'(x) = 1 for all x ≥ 1
(D) None of these

Q.10 If $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt$, then F'(4) equals-

(A) $\frac{32}{9}$ (B) $\frac{64}{3}$
(C) $\frac{64}{9}$ (D) None of these

MATHEMATICS IIT JEE (JULY 3rd WEEK CLASS TEST 5) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	C	C	C	B	B	B	B	D	A

SOLUTIONS

Sol.1 (A)

$\because$ $y = \frac{1}{12}x^3 \Rightarrow \frac{dy}{dx} = \frac{1}{4}x^2$

$\because$ x changes at faster rate than

$\Rightarrow \frac{dy}{dx} < 1$

$\Rightarrow \frac{x^2}{4} < 1$ or $x^2 < 4 \Rightarrow -2 < x < 2$

Sol.2 (C)

y = f(x) passes through (a, 0)

$\Rightarrow$ 0 = f(a) ......(1)

It has a unique tangent at (a, 0) means f'(a) exists.

Then $\lim_{x\to a} \frac{\log(1+6f(x))}{3f(x)}$ $\left(\because \frac{\log 1}{0} = \frac{0}{0}\right)$

$= \lim_{x\to a} \frac{\frac{1}{1+6f(x)}.6f'(x)}{3f'(x)}$ By L.H. Rule

$= \lim_{x\to a} \frac{2}{1+6f(x)} = \frac{2}{1+6f(a)} = 2$

Sol.3 (C)

$e^y = 1 + x^2 \Rightarrow y = \log(1 + x^2)$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x^2}$

i.e. slope of tangent $m = \frac{2x}{1+x^2}$

then $|m| = \frac{2|x|}{1+x^2}$

since A.M. $\geq$ G.M.

$\Rightarrow \frac{1+x^2}{2} \geq \sqrt{1.x^2}$

$\Rightarrow 1 + x^2 \geq 2|x|$

or $1 \geq \frac{2|x|}{1+x^2} \Rightarrow 1 \geq m$.

Sol.4 (C)

$f(x) = (x-1)^2|x-1| + x(x-1)^2$;

$(x-1)^2|x-1|$ is thrice differentiable in R – {1}.

$x(x-1)^2$ is thrice differentiable in R. Hence f(x) is thrice differentiable in R – {1}.

Sol.5 (B)

At x = 1 both curve touch each other

$\Rightarrow 1 + a - 1 = 6 + b$

$\Rightarrow a - b = 6$ .....(1)

and $\left(\frac{dy}{dx}\right)_{C_1}, \left(\frac{dy}{dx}\right)_{C_2}$ are equal

$\Rightarrow [3x^2 + a]_{x=1} = [12x]_{x=1}$

$\Rightarrow a = 9$ .....(2)

From (1) & (2) a = 9 and b = 3

Sol.6 (B)

When $x = \frac{\pi}{2}$, $y = \frac{2}{3}$, $\left[\frac{dy}{dx}\right]_{x=\pi/2}$

$= [\sin^3 x]_{x=\pi/2} = 1$

The equation of normal is

$y - \frac{2}{3} = -\left(x - \frac{\pi}{2}\right) \Rightarrow x + y = \frac{\pi}{2} + \frac{2}{3}$

Sol.7 (B)

$f(x) = x^{x^x} + x^{x^2} = e^{x^x \log x} + e^{x^2 \log x}$

$\Rightarrow f'(x) = x^{x^x}[x^x \log(ex)(\log x) + x^{x-1}] + (x^x)^x(2x\log x + x)$

$\Rightarrow f'(1) = 2$

Sol.8 (B)

(1, 1) lies on xy + ax + by = 0

$\Rightarrow a + b + 1 = 0$ (1)

$[dy/dx]_{(1,1)} = -(1 + a/1 + b) = 2$

$\Rightarrow a + 2b + 3 = 0$ (2)

from (1) and (2), we get a = 1, b = –2

Sol.9 (D)

We have,

$f(x) = \sqrt{(x-1)^2} = |x - 1|$

$$= \begin{cases} x-1, & \text{if } x \geq 1 \\ -(x-1), & \text{if } x < 1 \end{cases}$$

$$\therefore \quad f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$$

Sol.10 (A)

We have $F(x) = \dfrac{1}{x^2}\int_4^x (4t^2 - 2F'(t))\,dt.$

Therefore,

$$x^2 F(x) = \int_4^x (4t^2 - 2F'(t)\,dt$$

Differentiating both sides with respect to x, we get

$$2x\,F(x) + x^2 F'(x) = 4x^2 - 2F'(x)$$

Putting x = 4, we get

$8F(4) + 16F'(4) = 64 - 2F(4)$

$\Rightarrow \quad 18F'(4) = 64 \qquad [\because \quad F(4) = 0]$

$\Rightarrow \quad F'(4) = \dfrac{32}{9}$