Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,–1)(All questions have only one option correct).

Q.1 Consider the circle, $x^2 + y^2 = 9$. Let P be any point lying on the positive x-axis. Tangents are drawn from this point to the given circle, meeting the y-axis at P_1 and P_2 respectively. Find the coordinate of a point 'P' so that the area of the $\Delta P, P_1, P_2$ is minimum

(A) (3, 0) (B) $(3\sqrt{2}, 0)$

(C) $(\sqrt{2}, 0)$ (D) $(0, 3\sqrt{2})$

Q.2 If the equatin $e^{||x|-2|+b} = 2$ has four solution then b lies in :

(A) (log 2, – log 2) (B) (log 2 – 2, log 2)

(C) (–2, log 2) (D) (0, log 2)

Q.3 Find the point on the curve $3x^2 - 4y^2 = 72$ which is nearest to the line $3x + 2y + 1 = 0$

(A) (–6, 3) (B) (3, –6) (C) (3, 6) (D) (6, 3)

Q.4 If at each point of the curve $y = x^3 - ax^2 - x + 1$ the tangents is inclined at an acute angle with the positive direction of the x-axis, a lies in the interval.

(A) $[-\sqrt{3}, \sqrt{3})$ (B) $[-\sqrt{3}, \sqrt{3}]$

(C) $(-\sqrt{3}, \sqrt{3})$ (D) None of these

Q.5 The actue angle between the curves $y = |x^2 - 1|$ and $y = |x^2 - 3|$ at their points of intersection when x > 0

(A) $\tan^{-1}\frac{4\sqrt{2}}{7}$ (B) $\cot^{-1}\frac{4\sqrt{2}}{7}$

(C) $\tan^{-1}\frac{7}{4\sqrt{2}}$ (D) $\cot^{-1}\frac{7}{4\sqrt{2}}$

Q.6 The tangent represented by the graph of the function y = f(x) at the point with absissa x = 1 form an angle $\pi/6$ and at the point x = 2 an angle of $\pi/3$ and at the point x = 3 an agnle of $\pi/4$. Then the value of,

$$\int_1^3 f'(x)f''(x)\,dx + \int_2^3 f''(x)\,dx =$$

(A) $\frac{4-\sqrt{3}}{3}$ (B) $\frac{\sqrt{3}-4}{3}$

(C) $\frac{4-3\sqrt{3}}{3}$ (D) $\frac{-4-\sqrt{3}}{3}$

Let $f(x) = \int_0^{\sin x} t^2\,dt$ then answer the following questions from 7 to 9

Q.7 The interval in which sinx f'(x) increase in the interval $[0, \pi]$

(A) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (B) $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$

(C) $\left(\frac{\pi}{6}, \pi\right)$ (D) None of these

Q.8 The interval in which sin xf'(x) decreases in $[0, \pi]$ is

(A) $\left(\frac{\pi}{6}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{\pi}{6}, \frac{2\pi}{3}\right)$

(C) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (D) None of these

Q.9 The maximum value of sinx f'(x) in the interval $[0, \pi]$

(A) $\frac{3\sqrt{3}}{8}$ (B) $\frac{3\sqrt{3}}{16}$ (C) $\frac{3}{16}$ (D) None

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MATHEMATICS IIT JEE (AUGUST 3rd WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	B	B	A	B	A	C	B	C	B

SOLUTIONS

Sol.1 (B)

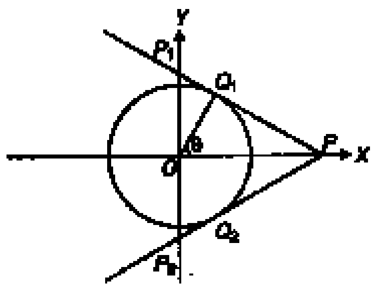
Let Q_1 and Q_2 be the points of contact.

Let OP = h and $\angle$QOP = θ

Clearly $OQ_1 = 3 \Rightarrow OP = h = 3\sec\theta$

And $OP_1 = 3\,\text{cosec}\,\theta$

Now, area of $\Delta\, PP_1P_2$



$$\Delta = \frac{1}{2}.(P_1P_2).(OP)$$

$$= (OP_1).(OP)$$

$$= 3\,\text{cosec}\,\theta \,.\, 3\sec\theta$$

$$\Delta = \frac{18}{\sin 2\theta}$$

Clearly Δ is minimum if $\sin 2\theta = 1$

$$\Rightarrow \theta = \frac{\pi}{4}$$

$$\Rightarrow h = 3\sec\left(\frac{\pi}{4}\right) = 3\sqrt{2}$$

Hence the required point is $P(3\sqrt{2}, 0)$.

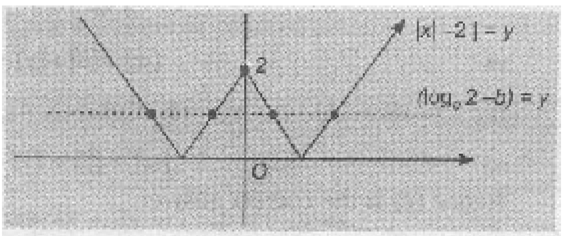
Sol.2 (B)

Here, $e^{||x|-2|+b} = 2$

$\Rightarrow ||x|-2| + b = \log_e 2$

$\Rightarrow ||x|-2| = \log_e 2 - b$

Clearly above equation has 4 roots if



$0 < (\log_e 2 - b) < 2$

$\Rightarrow -\log_e 2 < -b < 2 - \log_e 2$

$b \in (\log_e 2 - 2, \log_e 2)$

Sol.3 (A)

Slope of the given line 3x + 2y + 1 = 0 is (–3/2).

Let us locate the point on the curve at which the tangent is parrelell to given line.

Differentating the curve both sides with respect to x we get,

$$6x - 8y\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{3x_1}{4y_1} = \frac{-3}{2}$$ [since parallel to 3x + 2y = 1] also the point (x_1, y_1) lies on, $3x^2 - 4y^2 = 72$

$$3x_1^2 - 4y_1^2 = 72$$

$$\Rightarrow 3\frac{x_1^2}{y_1^2} - 4 = \frac{72}{y_1^2}$$

$$\Rightarrow 3(4) - 4 = \frac{72}{y_1^2} \quad \left[\text{as } \frac{x_1}{y_1} = 2\right]$$

$$\Rightarrow y_1^2 = 9$$

$$\Rightarrow y_1 = \pm 3$$

Required points are (–6, 3) and (6, –3)

Distanceof (–6, 3) from the given line,

$$= \frac{|-18+6+1|}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

and distance of (6, 3) from the given line,

$$= \frac{|-18-6+1|}{\sqrt{3}} = \frac{13}{\sqrt{3}} = \sqrt{13}$$

Sol.4 (B)

As, $y = x^3 - ax^2 + x + 1$

and the tangent is inclined at an acute angle with the positive direction of x-axis,

$$\Rightarrow \frac{dy}{dx} \ge 0$$

$\therefore\ 3x^2 - 2ax + 1 \ge 0$, for all $x \in R$

{and we know, $ax^2 + bx + c \ge 0$ for all $x \in R$

$\Rightarrow$ a > 0 and D $\le$ 0}

$\Rightarrow (2a)^2 - 4(3)(1) \le 0$

$\Rightarrow 4(a^2 - 3) \le 0$

$\Rightarrow (a - \sqrt{3})(a + \sqrt{3}) \le 0$

$\therefore -\sqrt{3} \le a \le \sqrt{3}$

Sol.5 (A)

For the intersection of the given curves

$|x^2 - 1| = |x^2 - 3| \Rightarrow (x^2 - 1)^2 = (x^2 - 3)^2$

$\Rightarrow (x^2 - 1)^2 - (x^2 - 3)^2 = 0$

$\Rightarrow [(x^2 - 1) - (x^2 - 3)][(x^2 - 1) + (x^2 - 3)] = 0$

$\Rightarrow 2[2x^2 - 4] = 0 \quad \Rightarrow 2x^2 = 4$

$\Rightarrow x = \pm\sqrt{2}$

neglecting $x = -\sqrt{2}$ as $x > 0$

We have point of intersection as $x = \sqrt{2}$

Here $y = |x^2 - 1| = (x^2 - 1)$ in the neighbouring of $x = \sqrt{2}$ and $y = -(x^2 - 3)$ in the neghbouring of $x = \sqrt{2}$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{C_1} = 2x = 2\sqrt{2}$$

$$\text{and} \quad \left(\frac{dy}{dx}\right)_{C_2} = -2x = -2\sqrt{2}$$

Hence, if θ is angle between them,

$$\Rightarrow \tan\theta = \left|\frac{2\sqrt{2} - (-2\sqrt{2})}{1 + 2\sqrt{2}(-2\sqrt{2})}\right| = \left|\frac{4\sqrt{2}}{-7}\right| = \left(\frac{4\sqrt{2}}{7}\right)$$

$$\therefore \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

Sol.6 (C)

Given, at $x = 1$, $\frac{dy}{dx} = \tan \pi/6 = 1/\sqrt{3}$

or at $x = 1 \Rightarrow f'(1) = \tan \pi/6 = 1/\sqrt{3}$

also at $x = 2 \Rightarrow f'(2) = \tan \pi/3 = \sqrt{3}$

and at $x = 3 \Rightarrow f'(3) = \tan \pi/4 = 1$

Then, $\int_1^3 f'(x)\, f''(x)\, dx + \int_2^3 f''(x)\, dx$

$$\Rightarrow \int_{f'(1)}^{f'(3)} t\, dt + \{f'(x)\}_2^3$$

Let $f'(x) = t$; $f''(x)\,dx = dt$

$$\Rightarrow \frac{1}{2}(t^2)_{f'(1)}^{f'(3)} + \{f'(3) - f'(2)\}$$

$$\Rightarrow \frac{1}{2}\{(f'(3))^2 - (f'(1))^2\} + \{f'(3) - f'(2)\}$$

$$\Rightarrow \frac{1}{2}\left\{(1)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right\} + \{1 - \sqrt{3}\}$$

$$\Rightarrow \frac{1}{2}\left(1 - \frac{1}{3}\right) + (1 - \sqrt{3})$$

$$\Rightarrow \frac{4}{3} - \sqrt{3} \quad \Rightarrow \frac{4 - 3\sqrt{3}}{3}$$

Sol.7 (B)

$g(x) = \sin x\, f'(x) = \sin^3 x \cos x$

$\Rightarrow g'(x) = \cos x\,(3\sin^2 x \cos x) + \sin^3 x\,(-\sin x)$

$= \sin^2 x\,(3\cos^2 x - \sin^2 x)$

$= \sin^2 x \cos^2 x\,(3 - \tan^2 x)$

Puttin $g'(x) = 0$ for which $g(x)$ to be increases

$\Rightarrow \tan^2 x = 3$; $x = n\pi \pm \frac{\pi}{3}$, $n \in N$

Putting $n = 0$; $x = \frac{\pi}{3}$, $x = -\frac{\pi}{3}$ rejected

$n = 1$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$ rejected

$\therefore$ Required points $\left(0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\right)$ are

Now intervals $0 < x < \frac{\pi}{3}$

Let $x = \pi/4 = 45^0$, $g'(x) = \sin^2 x\,(3\cos^2 x - \sin^2 x)$

$g'(\pi/4) = \frac{1}{2}\left(\frac{3}{2} - \frac{1}{2}\right)$, $g'\left(\frac{\pi}{4}\right) > 0$

$g(x)$ is increases in $0 < x < \pi/3$

$\frac{\pi}{3} < x < \frac{2\pi}{3}$; Let $x = 90^0$; $g'\left(\frac{\pi}{2}\right) = 1(0 - 1)$

$g'\left(\frac{\pi}{2}\right) < 0$ $\therefore g(x)$ decreases in $\frac{\pi}{3} < x < \frac{2\pi}{3}$

$\frac{2\pi}{3} < x < \pi$ let $x = 150^0$

$g'(150^0) = \frac{1}{4}\left(3.\frac{3}{4} - \frac{1}{4}\right) > 0$

$g(x)$ increasing in the interval $\frac{2\pi}{3} < x < \pi$

$\therefore g(x) = \sin x\, f'(x)$ increases

for $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$ and

Sol.8 (C)

Decreases in the interval $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$.

Sol.9 (B)