Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 If arg (z + a) = $\frac{\pi}{6}$ and arg (z – a) = $\frac{2\pi}{3}$; a $\in$ R^+, then-

(A) z is independent of a (B) |z| = |z+a|

(C) z = a cis $\frac{\pi}{6}$ (D) z = a cis $\frac{\pi}{3}$

Q.2 The origin and roots of the equation $z^2 + pz + q = 0$ form an equilateral triangle if-

(A) $p^2 = q$ (B) $p^2 = 3q$

(C) $q^2 = 3p$ (D) $q^2 = p$

Q.3 Number of complex numbers satisfying $z^3 = \bar{z}$ is-

(A) 1 (B) 2 (C) 4 (D) 5

Q.4 If z_1 and $\bar{z}_1$ represent adjacent vertices of a regular polygon of x sides with centre at the origin and if $\frac{\text{Im} z_1}{\text{Re} z_1} = \sqrt{2} - 1$ then the value of x is equal to-

(A) 8 (B) – 8 (C) 6 (D) 3

Q.5 The equation of the radical axis of the two circles represented by the equations |z – 2| = 3 and |z – 2 – 3i| = 4 on the complex plane is-

(A) 3y + 1 (B) 3y – 1

(C) 2y – 1 (D) None of these

Q.6 Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$ where p and q may be complex numbers. Let A and B represents z_1 and z_2 in the complex plane, given $\angle AOB = \alpha \neq 0$ and OA = OB. Where O is the origin using the given information what will be the value of p^2 ?

(A) 4q cos $\frac{\alpha}{2}$ (B) 4q sin $\frac{\alpha}{2}$

(C) 4q $\cos^2 \frac{\alpha}{2}$ (D) 4q $\sin^2 \frac{\alpha}{2}$

Q.7 If z_1, z_2, z_3 are the vertices of the ΔABC on the complex plane and are also the roots of the equation, $z^3 - 3\alpha z^2 + 3\beta z + x = 0$, then the condition for the ΔABC to be equilateral triangle is-

(A) $\alpha^2 = \beta$ (B) $\alpha = \beta^2$ (C) $\alpha^2 = 3\beta$ (D) $\alpha = 3\beta^2$

Q.8 If $p = a + b\omega + c\omega^2$, $q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where a, b, c $\neq$ 0 and ω is the complex cube root of unity, then -

(A) p + q + r = a + b + c

(B) $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$

(C) $p^2 + q^2 + r^2 = -2(pq + qr + rp)$

(D) None of these

Q.9 If $\alpha = e^{+2\pi/n}$, then $(11 - \alpha)(11 - \alpha^2)$ .... $(11 - \alpha^{n-1})$ is-

(A) 11^{n-1} (B) $\frac{11^{n-1}}{10}$

(C) $\frac{11^{n-1} - 1}{10}$ (D) $\frac{11^{n+1} - 1}{10}$

Q.10 Let z be a complex number having the argument θ, $0 < \theta < \frac{\pi}{2}$ and satisfying the equality |z – 3i| = 3 then $\cot \theta - \frac{6}{z}$ is equal to-

(A) 1 (B) – 1 (C) i (D) – i

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 4) (COMPLEX NUMBER) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	D	A	B	C	A	C	B	C

SOLUTIONS

Sol.1 (D)

$\arg(z + a) = \frac{\pi}{6}$, $\arg(z - a) = \frac{2\pi}{3}$

$\Rightarrow \arg((x + a) + iy) = \frac{\pi}{6}$

& $\arg((x - a) + iy) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\frac{y}{x+a} = \frac{\pi}{6}$, $\tan^{-1}\frac{y}{x-a} = \frac{2\pi}{3}$

$\Rightarrow \frac{y}{x+a} = \tan\frac{\pi}{6}$ and $\frac{y}{x-a} = \tan\frac{2\pi}{3}$

$\Rightarrow \frac{y}{x+a} = \frac{1}{\sqrt{3}}$ and $\frac{y}{x-a} = -\sqrt{3}$

$\Rightarrow \sqrt{3}\,y = x + a$ and $\sqrt{3}\,y = -3x + 3a$

On solving above equation

we get $x = \frac{a}{2}$ & $y = \frac{a\sqrt{3}}{2}$

Thus $z = \frac{a}{2} + \frac{ia\sqrt{3}}{2} = a\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$

$= a\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

Sol.2 (B)

$z^2 + pz + q = 0$ ..... (1)

given origin & roots of equation

i.e. $z_1 + z_2$ & z_1z_2 form an equilateral triangle

$\because\ z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_1z_3$

$\Rightarrow z_1^2 + z_2^2 = z_1z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$

from (1) $z_1 + z_2 = -p$, $z_1z_2 = q$

$\therefore\ (-p)^2 = 3q$

$\Rightarrow p^2 = 3q$

Sol.3 (D)

$z^3 = \bar{z}$ Let $z = x + iy$

$\Rightarrow (x + iy)^3 = \overline{x+iy}$

$\Rightarrow x^3 - iy^3 + 3x^2iy - 3xy^2 = x - iy$

$\Rightarrow x^3 - 3xy^2 = x$ and $3x^2y - y^3 = -y$

$\Rightarrow x^2 - 3y^2 = 1$ and $3x^2 - y^2 = -1$

On solving above equation, we get

$x^2 = -\frac{1}{6}$ and $y^2 = -\frac{1}{2}$, $x = 0$, $y = 0$

$\Rightarrow$ Number of complex numbers satisfying given equation are 5.

Sol.4 (A)

Let $z_1 = re^{i\theta}$ $\Rightarrow$ $\bar{z}_1 = re^{-i\theta}$

As z_1 and $\bar{z}_1$ are adjacent vertices of a regular polygon of n sides,

$\angle z_1 O \bar{z}_1 = \frac{2\pi}{n}$ and $|z_1| = |\bar{z}_1|$

i.e. $z_1 = \bar{z}_1\, e^{i2\pi/n}$

$\Rightarrow re^{i\theta} = re^{-i\theta}\, e^{i2\pi/n}$

$\Rightarrow \theta = \frac{2\pi}{n} - \theta$ $\Rightarrow$ $\theta = \frac{\pi}{n}$

$\therefore\ \frac{\text{Im}|z_1|}{\text{Re}|z_1|} = \sqrt{2} - 1$

$\Rightarrow \frac{r\sin\pi/n}{r\cos\pi/n} = \sqrt{2} - 1$

$\Rightarrow \tan\frac{\pi}{n} = \sqrt{2} - 1 = \tan\frac{\pi}{8}$

$\Rightarrow n = 8$

Sol.5 (B)

$\because$ equation of radical axis is given by $s_2 - s_1 = 0$

$\therefore\ |z - 2| = 3 \Rightarrow |(x - 2) + iy| = 3$

$\Rightarrow (x - 2)^2 + y^2 = 9$

$\Rightarrow s_1 = (x - 2)^2 + y^2 - 9 = 0$

Illy $|z - 2 - 3i| = 4$

$\Rightarrow |(x - 2) + i(y - 3)| = 4$

$\Rightarrow (x - 2)^2 + (y - 3)^2 = 16$

$\Rightarrow s_2 = (x - 2)^2 + (y - 3)^2 - 16 = 0$

Now $s_2 - s_1 = 0$

$\Rightarrow (y - 3)^2 - 16 - y^2 + 9 = 0$

$\Rightarrow -6y + 9 - 16 + 9 = 0$

$\Rightarrow -6y + 2 = 0 \Rightarrow 3y - 1 = 0.$

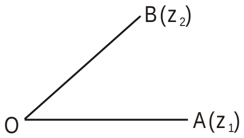
Sol.6 (C)

$\therefore \quad \vec{OB} = \vec{OA}\, e^{i\alpha}$

$(z_2 - 0) = (z_1 - 0)\, e^{i\alpha}$

$\frac{z_2}{z_1} = e^{i\alpha} = \cos\alpha + i\sin\alpha$

$\Rightarrow \frac{z_2}{z_1} = 2\cos^2\frac{\alpha}{2} - 1 + 2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$



$\Rightarrow \frac{z_2}{z_1} + 1 = 2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)$

$\Rightarrow \left(\frac{z_1 + z_2}{z_1}\right)^2 = 4\cos^2\frac{\alpha}{2}\left(\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}\right)^2$

$\Rightarrow \left(\frac{z_1 + z_2}{z_1}\right)^2 = 4\cos^2\frac{\alpha}{2}(\cos\alpha + i\sin\alpha)$

$= 4\cos^2\frac{\alpha}{2}\left(\frac{z_2}{z_1}\right)$

$\Rightarrow (z_1 + z_2)^2 = 4\cos^2\left(\frac{\alpha}{2}\right)(z_1 z_2)$ .... (1)

$\because$ from given equation

$z^2 + pz + q = 0$, $z_1 + z_2 = -p$, $z_1 z_2 = q$

$\therefore$ (2) becomes

$(-p)^2 = 4\cos^2\frac{\alpha}{2}q$

$\Rightarrow p^2 = 4q\cos^2\frac{\alpha}{2}$

Sol.7 (A)

Given z_1, z_2, z_3 are roots of equation $z^3 - 3\alpha z^2 + 3\beta z + x = 0$

$\Rightarrow z_1 + z_2 + z_3 = 3\alpha$

and $z_1z_2 + z_2z_3 + z_3z_1 = 3\beta$

$\because$ condition for z_1, z_2, z_3 to be equilateral triangle is

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

$\Rightarrow (z_1 + z_2 + z_3)^2 - 2(z_1z_2 + z_2z_3 + z_3z_1) = z_1z_2 + z_2z_3 + z_3z_1$

$\Rightarrow (3\alpha)^2 - 2(3\beta) = 3\beta$

$\Rightarrow 9\alpha^2 = 9\beta \quad \Rightarrow \quad \alpha^2 = \beta$

Sol.8 (C)

Given $p = a + b\omega + c\omega^2$, $q = b + c\omega + a\omega^2$, $r = c + a\omega + b\omega^2$

Now $p + q + r = a + b\omega + c\omega^2 + b + c\omega + a\omega^2 + c + a\omega + b\omega^2$

$= (a + b + c) + (a + b + c)\omega + (a + b + c)\omega^2$

$= (a + b + c)(1 + \omega + \omega^2) = 0$

$\therefore \quad p + q + r = 0$

$\Rightarrow p^2 + q^2 + r^2$

$= (p + q + r)^2 - 2(pq + qr + rp)$

$= 0 - 2(pq + qr + rp)$

$= -2(pq + qr + rp)$

Sol.9 (B)

Given $\alpha = e^{\frac{i2\pi}{n}}$

$\Rightarrow \alpha$ is n, n^{th} root of unity

$\Rightarrow (z - \alpha)(z - \alpha^2)(z - \alpha^3)$ ....

$(z - \alpha^{n-1}) = \frac{z^n - 1}{z - 1}$

Now $(11 - \alpha)(11 - \alpha^2)$ ..... $(11 - \alpha^{n-1})$

$\Rightarrow 11^n - 1 = (11 - 1)(11 - \alpha)(11 - \alpha^2)$ .... $(11 - \alpha^{n-1})$

$\Rightarrow (11 - \alpha)(11 - \alpha^2)$ ...... $(11 - \alpha^{n-1})$

$= \frac{11^{n-1}}{11 - 1} = \frac{11^{n-1}}{10}$

Sol.10 (C)

Let $z = re^{i\theta}$

$\therefore \quad z - 3i = r\cos\theta + i(r\sin\theta - 3)$

Given $|z - 3i| = 3$

$\Rightarrow r^2\cos^2\theta + (r\sin\theta - 3)^2 = 3^2$

$\Rightarrow r^2 - 6r\sin\theta = 0$

$\Rightarrow r = 0$ or $r = 6\sin\theta$

Now, $\cot\theta - \frac{6}{z}$

$= \cot\theta - \frac{6}{re^{i\theta}}$

$= \cot\theta - \frac{6}{6\sin\theta\, e^{i\theta}}$

$= \frac{\cos\theta}{\sin\theta} - \frac{e^{-i\theta}}{\sin\theta} = \frac{\cos\theta - (\cos\theta - i\sin\theta)}{\sin\theta}$

$= i$