Dear student following is a Moderate level [○●○] test paper. Score of 12 Marks in 15 Minutes would be a satisfactory performance. Questions 1-8(+3,−1)(All questions have only one option correct).

Q.1 Maximum area of a triangle inscribed in an ellipse so that extremity of major axis is one of the vertices of the triangle is-

(A) $\frac{\sqrt{3}ab}{4}$ (B) $\frac{3ab}{4}$

(C) $\frac{3\sqrt{3}ab}{4}$ (E) $\frac{3\sqrt{3}}{4ab}$

Q.2 The tangent and the normal drawn to the curve $y = x^2 - x + 4$ at P(1, 4) cut the X-axis at A and B respectively. If the length of the subtangent drawn to the curve at P is equal to the length of the subnormal, then the area of the triangle PAB in sq. units is-

(A) 4 (B) 32 (C) 8 (D) 16

Q.3 If f(x) and g(x) are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$, then f'(x) =

(A) $3x^2 + 3$ (B) $x^2 - \frac{1}{x^2}$

(C) $1 + \frac{1}{x^2}$ (D) $3x^2 + \frac{3}{x^4}$

Q.4 If f(x) is a function such that $f''(x) + f(x) = 0$ and $g(x) = [f(x)]^2 + [f'(x)]^2$ and $g(3) = 3$ then g(8) =

(A) 5 (B) 0 (C) 3 (D) 8

Q.5 f(x) is cubic polynomial which has local maximum at x = − 1. If f(2) = 18, f(1) = − 1 and f'(x) has local minima at x = 0, then-

(A) The distance between (−1, 2) and (a, f(a)), where x = a is the point of local minima is $2\sqrt{5}$

(B) f(x) is increasing for $x \in [1, 2\sqrt{5}]$

(C) f(x) has local maxima at x = 1

(D) The value of f(0) is 5.

Q.6 If $f''(x) < 0 \ \forall \ x \in (a, b)$ and c is a point such that a < c < b, and (c, f(c)) is the point lying on the curve for which F(c) is maximum, then f'(c) is equal to-

(A) $\frac{f(b) - f(a)}{b - a}$ (B) $\frac{2(f(b) - f(a))}{b - a}$

(C) $\frac{2f(b) - f(a)}{2b - a}$ (D) 0

Ram draw a rectangle having two vertices on the positive x-axis and other two vertices on the lines y = x and y = − 5x + 6. Then he draw another rectangle having two vertices on the positive y-axis and other two vertices on the same above two lines.
Answer the following equations -

Q.7 What is the maximum possible area of rectangle in first case ?

(A) 4/5 (B) 1/10 (C) 2/5 (D) 3/10

Q.8 What is the maximum possible area of rectangle in second case ?

(A) 1/2 (B) 3/2 (C) 5/2 (D) 3/5

MATHEMATICS IIT JEE (AUGUST 3rd WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

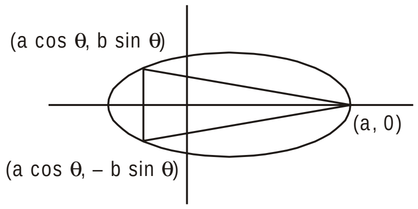
Que.	1	2	3	4	5	6	7	8
Ans.	C	D	A	C	B	A	D	B

SOLUTIONS

Sol.1 (C)

A(a, 0), B(a cos θ, b sin θ),
C(a cos θ – b sin θ)

Where $\pi > \theta > \frac{\pi}{2}$



Area of ΔABC

$= b \sin\theta\,(a - a\cos\theta)$

$= ab\left(\sin\theta - \frac{\sin 2\theta}{2}\right)$

$f'(\theta) = 0$ gives

$\cos\theta = \cos 2\theta$

$\Rightarrow\ \theta = 120^o$

$\therefore$ Max. area = ab (sin 120º) (1 – cos 120º)

$= \frac{\sqrt{3}ab}{2}\ \frac{3}{2} = \frac{3\sqrt{3}ab}{4}$

Sol.2 (D)

Given curve $y = x^2 - x + 4$
Slope of tangent at P(1, 4) is

$\left(\frac{dy}{dx}\right)_p = (2x-1)_{(1,\,4)} = 1$

$\therefore$ Equation of tangent

$y - 4 = 1(x - 1) = 3$ .....(1)

Similarly, slope of normal $= -\frac{1}{dy/dx} = -1$

$\therefore$ Equation of normal is

$y - 4 = -1(x - 1)$

$\Rightarrow$ x + y = 5 ....(2)

Tangent cuts x-axis at A

$\therefore$ A(–3, 0)

and normal cuts y-axis at B.

$\therefore$ B(5, 0)

$\therefore$ Area of ΔPAB $= \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ -3 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}|\,4(-3-5)| = 16$ sq. units.

Sol.3 (A)

$fog(x) = x^3 - \frac{1}{x^3}$

writing $x^3 - \frac{1}{x^3}$ using

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$,

we have $\left(x^3 - \frac{1}{x^3}\right) = x^3 - \frac{1}{x^3} - 3x.\frac{1}{x}\left(x - \frac{1}{x}\right)$

$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

We have

$f(g(x)) = x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

As $g(x) = x - \frac{1}{x}$, this yields

$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$

Setting $x - \frac{1}{x} = t$, $f(t) = t^3 + 3t$

Thus $f(x) = x^3 + 3x$, $f'(x) = 3x^2 + 3$.

Sol.4 (C)

$g(x) = [f(x)]^2 + [f'(x)^2]$

Differentiating w.r.t. x

$g'(x) = 2f(x)\,f'(x) + 2f'(x)\,f''(x)$

$= 2f'(x)\,[f(x) + 2f''(x)] = 2f'(x).0 = 0$

(By hypothesis $f(x) + f''(x) = 0$)

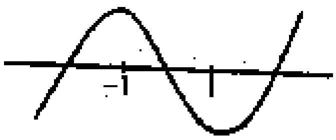
$\therefore$ g(x) = constant.

Thus g(8) = g(3) = 3.

Sol.5 (B)

The required polynomial which satisfy the condition is $f(x) = \frac{1}{4}(19x^3 - 57x + 34)$

f(x) has local maximum at x = – 1 and local minimum at x = 1



Hence f(x) is increasing for $x \in \left[1, 2\sqrt{5}\right]$

Sol.6 (A)

$F'(c) = (b - a)\, f'(c) + f(a) - f(b)$

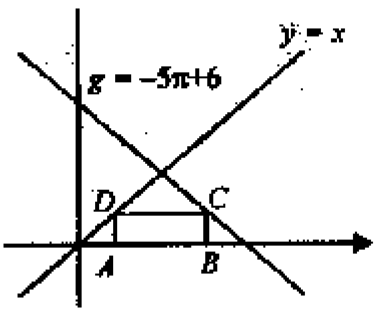
$F''(c) = f''(c)\,(b - a) < 0$

$\Rightarrow \quad F'(c) = 0 \rightarrow f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Sol.7 (D)

Let x-coordinate of points A and B are x_1 and x_2 respectively.

Then BC = $-5x_2 + 6$ and AD = x_1



AD and BC arc opposite sies of a rectangle

$\Rightarrow \quad -5x_2 + 6 = x_1 \qquad \Rightarrow \qquad x_2 = \dfrac{6 - x_1}{5}$

Area of rectangle, A = (AD) × (AB)

$A = x_1(x_2 - x_1)$

$\Rightarrow \quad A = x_1\left(\dfrac{6 - 6x_1}{5}\right);$

$\Rightarrow \quad \dfrac{dA}{dx_1} = \dfrac{6}{5} - \dfrac{12}{5}x_1$

$\dfrac{dA}{dx_1} = 0 \Rightarrow x_1 = \dfrac{1}{2}$

$\dfrac{d^2A}{dx_1^2} = \dfrac{-12}{5} < 0$

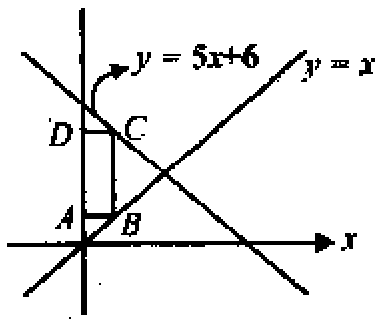
Area = $1/2\left[\dfrac{6 - 6/2}{5}\right]$ = 3/10.

Sol.8 (B)

Let point A(0, y_1) and D(0, y_2)

$\Rightarrow \quad AB = y_1$

$CD = \dfrac{y_2 - 6}{-5}$; AB = CD



$\Rightarrow \quad y_1 = \dfrac{y_2 - 6}{-5} \qquad \Rightarrow \qquad -5y_1 = y_2 - 6$

$\Rightarrow \quad y_2 = 6 - 5y_1$

Area, A = AD × AB

$= (y_2 - y_1) \times (y_1) = (6 - 5y_1 - y_1)y_1$

$A = (6 - 6y_1)y_1$

$\dfrac{dA}{dy_1} = 6 - 12y_1; \quad \dfrac{dA}{dy_1} = 0$

$\Rightarrow \quad y_1 = 1/2$

$\dfrac{d^2A}{dy_1^2} = -12 < 0;$

$\therefore \quad$ Max. Area = (6 – 6/2)1/2 = 3/2