Dear student following is a Moderate level [○●○] test paper. Score of 12 Marks in 15 Minutes would be a satisfactory performance. Questions 1-7(+3,−1)(All questions have only one option correct).

Q.1 The minimum intercept made by the axes on the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is-

(A) $a^2 + b^2$ (B) $a + b$
(C) $a - b$ (D) None of these

Q.2 The least value of a for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = a$ has at least one solution on the interval $\left(0, \frac{\pi}{2}\right)$ is-

(A) 9 (B) 4
(C) 8 (D) 1

Q.3 The sides of the rectangle of greatest area which can be inscribed in the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ are given by-

(A) $(4, 2\sqrt{2})$ (B) $(2\sqrt{2}, 2)$
(C) $(2, \sqrt{2})$ (D) $4\sqrt{2}, 4)$

A man walks in a horizontal circle round the foot of a pole which is inclined to the vertical. The foot of the pole is at the centre of the circle. The greatest and the least angles which the pole subtends at his eyes are $\tan^{-1}\frac{9}{5}$ and $\tan^{-1}\frac{6}{5}$ respectively, and when he is midway between the corresponding positions, the angle is θ. If the man's height be neglected, find the length of the pole and θ, if the radius of the circle is a, and the length of the pole > a.

Q.4 What is the length of the pole ?

(A) $\frac{\sqrt{1321}}{25}a$ (B) $\frac{\sqrt{1321}}{5}a$
(C) $\frac{\sqrt{1321}}{36}a$ (D) $\frac{\sqrt{1321}}{6}a$

Q.5 What is the inclination of the pole with vertical ?

(A) $\tan^{-1} 5/36$ (B) $\tan^{-1}\frac{25}{36}$
(C) $\tan^{-1} 5/6$ (D) $\tan^{-1}\left(\frac{25}{\sqrt{1321}}\right)$

Q.6 What is the value of $\tan\theta$?

(A) $\frac{\sqrt{1321}}{36}$ (B) $\frac{\sqrt{1321}}{25}$
(C) $\frac{\sqrt{1321}}{6}$ (D) $\frac{\sqrt{1321}}{5}$

Q.7 How much distance the man walks when we goes from the position of least angle ($\tan^{-1} 6/5$) to the position of greatest angle ($\tan^{-1} 9/5$) ?

(A) $\frac{3}{4}\pi a$ (B) $\frac{\pi}{2}a$
(C) πa (D) $\frac{\pi}{4}a$

MATHEMATICS IIT JEE (AUGUST 3rd WEEK CLASS TEST 3) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○					
3	○	○	○	○	6	○	○	○	○					

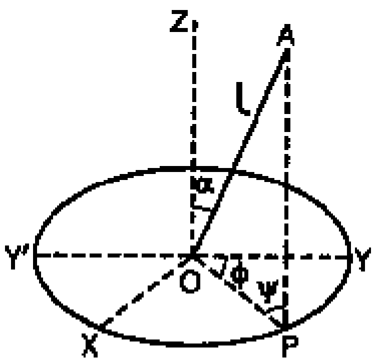
ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	B	A	A	A	A	B	C

SOLUTIONS

Sol.4-7

Let O be the centre of the circle and the pole OA be inclined to the vertical OZ at an angle α. Let OA = l.



Let the plane of OZ and OA cut the circle at Y and Y', Y being closer to A. OX $\perp$ OY in the plane of the circle.

Take OX, OY and OZ as the x, y and z axes.

P be any point on the circle. Let $\angle POY = \phi$ and $\angle OPA = \psi$.

Now, A = (0, lsin α, lcos α) and P = (asin ϕ, acos ϕ, 0)

where the radius = a

$\therefore \quad \vec{OP} = a\sin\phi\,\vec{i} + a\cos\phi\,\vec{j}$

$\vec{OA} = l\sin\alpha\,\vec{j} + l\cos\alpha\,\vec{k}$

$\vec{AP} = \vec{OP} - \vec{OA}$

$= a\sin\phi\,\vec{i} + (a\cos\phi - l\sin\alpha)\,\vec{j} - l\cos\alpha\,\vec{k}$

$\therefore \quad AP = |\vec{AP}|$

$$= \sqrt{a^2\sin^2\phi + (a\cos\phi - l\sin\alpha)^2 + l^2\cos^2\alpha}$$

$$= \sqrt{a^2 + l^2 - 2al\cos\phi.\sin\alpha}$$

$$\therefore \quad \cos\psi = \frac{OP^2 + AP^2 - OA^2}{2OP.AP}$$

$$= \frac{a^2 + a^2 + l^2 - 2al\cos\phi.\sin\alpha - l^2}{2a\sqrt{a^2 + l^2 - 2al\cos\phi.\sin\alpha}}$$

$$= \frac{a - l\cos\phi.\sin\alpha}{\sqrt{a^2 + l^2 - 2al\cos\phi.\sin\alpha}}$$

$\therefore \quad \tan\psi$ =

$$\frac{\left\{a^2 + l^2 - 2al\cos\phi\sin\alpha - (a - l\cos\phi.\sin\alpha)^2\right\}^{1/2}}{a - l\cos\phi.\sin\alpha}$$

$$= \frac{\sqrt{l^2 - l^2\cos^2\phi.\sin^2\alpha}}{a - l\cos\phi.\sin\alpha} \quad ...(1)$$

ψ and so tan ψ is maximum or minimum when $\frac{d(\tan\psi)}{d\phi} = 0$

or $\frac{1.(2l^2\cos\phi.\sin\phi.\sin^2\alpha)}{2\sqrt{l^2 - l^2\cos^2\phi.\sin^2\alpha}}$. (a - lcos ϕ . sin α) - $\sqrt{l^2 - l^2\cos^2\phi.\sin^2\alpha}$ (lsin ϕ sin α) = 0

or $l^2\cos\phi \,.\, \sin\phi\,\sin^2\alpha(a - l\cos\phi\sin\alpha) - (l^2 - l^2\cos^2\phi \,.\, \sin^2\alpha)\, l\sin\phi\sin\alpha = 0$

or $\cos\phi.\ \sin\phi\ \sin\alpha(a - l\cos\phi\ \sin\alpha)$
$- (1 - \cos^2\phi.\ \sin^2\alpha)\ l\sin\phi = 0$

or $\sin\phi\ [a\cos\phi\sin\alpha - l\cos^2\phi\sin^2\alpha$
$- l + l\cos^2\phi.\ \sin^2\alpha] = 0$

or $\sin\phi[a\cos\phi\sin\alpha - l] = 0$

$\therefore$ sin ϕ = 0, i.e., ϕ = 0 or π

or acos ϕsin α = l,

i.e., cos ϕ = $\frac{l}{a\sin\alpha}$ (not possible)

$\therefore$ ψ is maximum when ϕ = 0 and it is minimum when $\phi = \pi$.

When ϕ = 0, from (1),

$$\tan\psi = \frac{9}{5} = \frac{\sqrt{l^2 - l^2\sin^2\alpha}}{a - l\sin\alpha} = \frac{l\cos\alpha}{a - l\sin\alpha}$$

$\therefore$ 9a - 9l sin α = 5lcos α

or 9a = l(5cos α + 9sin α) ...(2)

When $\phi = \pi$, from (1),

$$\tan\psi = \frac{6}{5} = \frac{\sqrt{l^2 - l^2\sin^2\alpha}}{a + l\sin\alpha} = \frac{l\cos\alpha}{a + l\sin\alpha}$$

$\therefore$ 6a + 6lsin α = 5l cos α

or 6a = l(5cos α - 6sin α) ...(3)

When the man is midway, $\phi = \frac{\pi}{2}$. So, from(1),

$$\tan\theta = \frac{l}{a} \qquad ...(4)$$

From (2) and (3),

$$\frac{3}{2} = \frac{5\cos\alpha + 9\sin\alpha}{5\cos\alpha - 6\sin\alpha}$$

or 15cos α - 18sin α = 10cos α + 18sin α or 5cos α = 36sin α

$\therefore$ tan α = $\frac{5}{36}$.

$\therefore$ cos α = $\frac{36}{\sqrt{1321}}$, sin α = $\frac{5}{\sqrt{1321}}$

$$\therefore (3) \Rightarrow l = \frac{6a}{5\cos\alpha - 6\sin\alpha}$$

$$= \frac{6a}{\frac{5\times36 - 6\times5}{\sqrt{1321}}} = \frac{\sqrt{1321}}{25}a.$$

$\therefore$ by (4), tan θ = $\frac{\sqrt{1321}}{25}$

$$\therefore \theta = \tan^{-1}\frac{\sqrt{1321}}{25}.$$