Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 A man of height 6 ft is moving away from a lamp post of height 10ft, at the rate of 4ft/second then the rate at which his shadow will increase is-

(A) 2 ft/sec (B) 4 ft/sec
(C) 6 ft/sec (D) None of these

Q.2 If $f(x) = \frac{x}{\sin x}$, $g(x) = \frac{x}{\tan x}$ $0 < x \le 1$, then

(A) Both f(x) and g(x) are increasing
(B) Both are decreasing
(C) f(x) is increasing (D) g(x) is decreasing

Q.3 If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$ then $\frac{dy}{dx}$ equals

(A) 0 (B) – 1
(C) 1 D) $\frac{-1}{\sqrt{1-(x+2/\sqrt{13})^2}}$

Q.4 The function $x^{25}(1-x)^{75}$ assumes maximum value for

(A) x = 0 (B) x = 1/4
(C) x = 3/4 (D) None of these

Q.5 The co-ordinates of the point P on the curve $y^2 = 2x^3$, the tangent at which is perpendicular to the line 4x – 3y + 2 = 0, are given by :

(A) (2, 4) (B) $(1, \sqrt{2})$
(C) (1/2, – 1/2) (D) (1/8, –1/16)

Q.6 If $f(x) = a\log|x| + bx^2 + x$ has extremums at x = 1 and x = 3, then

(A) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (B) $a = \frac{3}{4}, b = -\frac{1}{8}$
(C) $a = -\frac{3}{4}, b = \frac{1}{8}$ (D) None of these

Q.7 The chord joining the points x = p, x = q on the curve $y = ax^2 + bx + c$ is parallel to the tangent at the point on the curve whose abscissa is

(A) $\frac{1}{2}(p + q)$ (B) $\frac{1}{2}(p - q)$
(C) $\frac{1}{2}pq$ (D) None of these

Q.8 The point (0, 3) is nearest to the curve $x^2 = 2y$ at

(A) $(2\sqrt{2}, 0)$ (B) (0, 0)
(C) (2, 2) (D) None of these

Q.9 If there in an error of K % in measuring the edge of a cube then the percentage error in estimating its volume is

(A) K (B) 3K
(C) K/3 (D) None

Q.10 A differentiable function f(x) has a relative minima at x = 0 then the function y = f(x) + ax + b has a relative minima at x = 0 for

(A) All a and all b (B) All b if a = 0
(C) All b > 0 (D) All a > 0

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	C	B	D	A	A	C	B	B

SOLUTIONS

Sol.1 (C)

OA = 10ft is lamp post BD = 6ft is man.
Let at any time t man has moved through a distance y and x be the length of it's shadow.
Then in similar ΔBDC and ΔAOC we have

$$\frac{x+y}{AO} = \frac{x}{BD} \Rightarrow \frac{x+y}{10} = \frac{x}{6}$$

$$\Rightarrow \quad 4x = 6y \Rightarrow \frac{dx}{dt} = \frac{3}{2}\frac{dy}{dt}$$

But $\frac{dy}{dt}$ = speed of man = 4 ft/second.

Then rate of increasing of shadow

$$\frac{dx}{dt} = \frac{3}{2} \times 4 = 6 \text{ ft/s.}$$

Sol.2 (C)

$$f(x) = \frac{x}{\sin x} \qquad \Rightarrow f'(x) = \frac{\sin x - x\cos x}{\sin^2 x}$$

And $\frac{d}{dx}$ (sinx – x cos x) = x sin x $\geq$ 0 for all $x \in [0, 1]$.

Thus sin x – x cos x is an increasing function in [0, 1].
Then for 0 < x < 1 we have
sin 0 – 0 cos0 < sinx – x cosx
or sinx – x cosx > 0 $x \in [0, 1]$

Thus f'(x) $\frac{\sin x - x\cos x}{\sin^2 x}$ > 0 whenever

$0 < x \leq 1$. Thus f(x) is increasing in this interval.

Again $g(x) = \frac{x}{\tan x}$

$$\Rightarrow \quad g'(x) = \frac{1}{\tan x} - \frac{x\sec^2 x}{\tan^2 x}$$

$$\Rightarrow \quad g'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x}$$

And $\frac{d}{dx}$(tanx – x sec^{2}x) = –2sec^{2}x tanx < 0 $x \in [0, 1]$

Thus tanx – xsec^{2}x is decreasing function on [0, 1]
Then for 0 < x < 1, tan 0 – 0 sec^2 0 > tan x – x sec^2 x
tan x – x sec^2 x < 0
whenever 0 < x< 1.

Then $g'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x}$ < 0 whenever 0 < x < 1. Thus g(x) is decreasing.

Sol.3 (C)

$$y = \cos^{-1}\left[\frac{2}{\sqrt{13}}\cos x - \frac{3}{\sqrt{13}}\sin x\right]$$

$= \cos^{-1}$ [cosx cosθ –sinx sinθ]

Where $\cos\theta = \frac{2}{\sqrt{13}}$, $\sin\theta = \frac{3}{\sqrt{13}}$

$= \cos^{-1}$ [cos(x + θ)] = x + θ

$$\Rightarrow \frac{dy}{dx} = 1.$$

Sol.4 (B)

Let $y = x^{25}(1-x)^{75}$

$$\Rightarrow \quad \frac{dy}{dx} = 25x^{24}(1-x)^{75} - 75(1-x)^{74}x^{25}$$

$$= 25x^{24}(1-x)^{74}[(1-x) - 3x]$$

$$\frac{dy}{dx} = 25x^{24}(1-x)^{74}(1-4x)$$

$$= 100x^{24}(1-x)^{74}\left(\frac{1}{4} - x\right)$$

Now $\frac{dy}{dx} = 0 \quad \Rightarrow x = 0, 1, 1/4$

We also observe that sign of $\frac{dy}{dx}$ will remain unchanged as x crosses 0 and 1, but sign of $\frac{dy}{dx}$ changes from + ive to negative as x crosses 1/4 from left to right. Thus maximum occurs at 1/4.

Sol.5 (D)

Slope of given line is m = $\frac{4}{3}$

Given equation of curve $y^2 = 2x^3$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$

Slope of tangent at (x, y) is $m_1 = \frac{3x^2}{y}$

Tangent will be $\perp$ to given line if

$m\,m_1 = -1 \quad \Rightarrow \frac{4}{3} . \frac{3x^2}{y} = -1$

$\Rightarrow y = -4x^2$

On putting this in eq. of curve we get

$(-4x^2)^2 = 2x^3 \quad \Rightarrow \quad x^3(16x - 2) = 0$

$\Rightarrow x = \frac{1}{8}$ then $y = -4x^2 = -4.\frac{1}{64} = -\frac{1}{16}$

Point is $\left(\frac{1}{8}, -\frac{1}{16}\right)$

Sol.6 (A)

In the neighbourhood of x = 1 and x = 3 we have |x| = x, thus

$f(x) = a\log x + bx^2 + x$

$\Rightarrow f'(x) = \frac{a}{x} + 2bx + 1$

Now f(x) has extremums at x = 1 and 3 then f'(1) = f'(3) = 0

$\Rightarrow f'(1) = \frac{a}{1} + 2b + 1 = 0$

or $a + 2b + 1 = 0$

and $f'(3) = \frac{a}{3} + 6b + 1 = 0$ or $a + 18b + 3 = 0$

Solving we get $b = -\frac{1}{8}$, $a = -\frac{3}{4}$

Sol.7 (A)

At x = p, $y_p = ap^2 + bp + c$ and at x = q

$y_q = aq^2 + bq + c$

Slope of chord joining points x = p and x = q is

$m = \frac{y_q - y_p}{q - p} = \frac{a(q-p)(q+p) + b(q-p)}{q-p}$

$= a(p + q) + b$

Slope of tangent $\frac{dy}{dx} = 2ax + b$ then at the point where tangent is parallel to chord joining points x = q, x = p we have

$2ax + b = a(p + q) + b$

$x = \frac{1}{2}(p + q)$.

Sol.8 (C)

Let (x, y) be any point on $x^2 = 2y$ It's distance from (0, 3) is

$d = \sqrt{(x-0)^2 + (y-3)^2}$

$= \sqrt{x^2 + (y-3)^2}$

Let $z = d^2 = x^2 + (y - 3)^2$

$= 2y + (y - 3)^2 \qquad$ As $x^2 = 2y$

$z = y^2 - 4y + 9 = (y - 2)^2 + 5$

This will be least for y = 2

When y = 2, $x^2 = 2y = 4 \quad \Rightarrow \quad x = 2$

Thus point (0, 3) is nearest to curve at (2, 2).

Sol.9 (B)

Let a be the side of cube then volume $V = a^2$

$\Rightarrow \frac{dV}{da} = 3a^2 \qquad$ or $dV = \frac{3a^2}{a^3} \times a^3 da$

or $dV = \frac{3}{a} V da \qquad$ or $\frac{dV}{V} = 3\frac{da}{a}$

or $\frac{dV}{V} \times 100 = 3\frac{da}{a} \times 100$

$\Rightarrow \frac{dV}{V} \times 100\% = 3 \times k\% \left(\because \frac{da}{a} \times 100 = k\right)$

$= 3k\,\%$

Sol.10 (B)

Given that y = f(x) + ax + b

Since f(x) has minima at x = 0

$\Rightarrow f'(0) = 0$ and $f''(0) < 0$

Further $y' = f'(x) + a$

and $(y'')_0 = f''(0) < 0$

Now y will also have a minima at x = 0 if

$(y')_{x=0} = 0 \qquad \Rightarrow f'(0) + a = 0 \Rightarrow a = 0$