Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (Questions may have more than one option correct).

Q.1 let f(x) and g(x) be two functions having finite non-zero 3rd order derivatives f'''(x) and g'''(x) for all $x \in R$. If f(x) g(x) = 1 for all $x \in R$, then $\frac{f'''}{f'} - \frac{g'''}{g'}$ is equal to

(A) $3\left(\frac{f'}{g} - \frac{g'}{f}\right)$ (B) $3\left(\frac{f'}{f} - \frac{g'}{g}\right)$

(C) $3\left(\frac{g'}{g} - \frac{f'}{g}\right)$ (D) $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$

Q.2 If $y^2 = P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$ is equal to

(A) P(x) + P"(x) (B) P(x)

(C) P(x) P'''(x) (D) A constant.

Q.3 The derivative of $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

(A) $-\frac{1}{2\sqrt{1-x^2}}$ (B) $\frac{1}{2\sqrt{1-x^2}}$

(C) $\frac{2}{\sqrt{1-x^2}}$ (D) $\frac{-2}{\sqrt{1-x^2}}$

Q.4 If the line ax + by + c = 0 is normal to xy = 1, then

(A) a > 0, b > 0 (B) a > 0, b < 0

(C) b > 0, a < 0 (D) a < 0, b < 0.

Q.5 The equation of the tangents at the origin to the curve. $y^2 = x^2(1 + x)$ are

(A) $y = \pm x$ (B) $x = \pm y$

(C) $y = \pm 2x$ (D) None of these

Q.6 The length of the subtangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point (4, 1) is

(A) 2 (B) 1/2

(C) 3 (D) 4.

Q.7 Let f be differentiable for all x. If f(1) = –2 and $f'(x) \geq 2$ for all $x \in [1, 6]$, then

(A) f(6) < 8 (B) $f(6) \geq 8$

(C) $f(6) \geq 5$ (D) $f(6) \leq 5$.

Q.8 If the function f(x) = cos |x| – 2ax + b increases along the entire number scale, the range of values of a is given by

(A) $a \leq b$ (B) $a = \frac{b}{2}$

(C) $a \leq -\frac{1}{2}$ (D) $a \geq -\frac{3}{2}$

Q.9 The value of a in order that

$f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$

decreases for all real values of x, is given by

(A) a < 1 (B) $a \geq 1$

(C) $a \geq \sqrt{2}$ (D) $a < \sqrt{2}$

Q.10 If a < 0, the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for values of x given by

(A) x > 0 (B) x < 0

(C) x > 1 (D) x < 1

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	A	B,C	A	B	B	C	B	B

SOLUTIONS

Sol.1 (B)

We have f(x) g(x) = 1. Differentiating with respect to x, we get

$f'g + f g' = 0$ .... (1)

Differentiating (1) w.r.t. x, we get

$f''g + 2f'g' + fg'' = 0$ .... (2)

Differentiating (2) w.r.t. x we get

$f'''g + g''' f + 3f''g' + 3g'' f' = 0$

$$\Rightarrow \frac{f'''}{f'}(f'g) + \frac{g'''}{g'}(fg') + \frac{3f''}{f}(fg') + \frac{3g''}{g}(gf') = 0$$

$$\Rightarrow \left(\frac{f'''}{f'} + \frac{3g''}{g}\right)(f'g) = -\left(\frac{g'''}{g'} + \frac{3f''}{f}\right)(fg')$$

$$\Rightarrow -\left(\frac{f'''}{f'} + \frac{3g''}{g}\right)(fg') = -\left(\frac{g'''}{g'} + \frac{3f''}{f}\right)(fg')$$

$$\Rightarrow \frac{f'''}{f'} + \frac{3g''}{g} = \frac{g'''}{g'} + \frac{3f''}{f}$$

$$\Rightarrow \frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} + \frac{g''}{g}\right)$$

Sol.2 (C)

We have , $y^2 = P(x) \Rightarrow 2yy_1 = P'(x)$

$$\Rightarrow 2y_1 = \frac{P'(x)}{y}$$

$$\Rightarrow 2y_2 = \frac{yP''(x) - P'(x)y_1}{y^2}$$

$$\Rightarrow 2y_2 = \frac{yP''(x) - \frac{P'(x)P'(x)}{2y}}{y^2}$$

$$= \frac{2y^2P''(x) - (P'(x))^2}{2y^3}$$

$$\Rightarrow 2y^3y_2 = \frac{1}{2}[2y^2 P''(x) - (P'(x))^2]$$

$$\Rightarrow 2y^3y_2 = \frac{1}{2}[2 P(x) P''(x) - (P'(x))^2]$$

$$\Rightarrow 2\frac{d}{dx}(y^3y_2)$$

$$= \frac{1}{2}[2P'(x)P''(x) + 2P(x)P'''(x) - 2P'(x)P''(x)]$$

$$= P(x) P'''(x)$$

Sol.3 (A)

Let $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$. Putting x = cosθ. we get

$$y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}\right)$$

$$= \sin^{-1}\left(\sin\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Sol.4 (B,C)

Any point on xy = 1 is $\left(t, \frac{1}{t}\right)$, $t \neq 0$

Now, xy = 1 $\Rightarrow x\frac{dy}{dx} + y = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \left(\frac{dy}{dx}\right)_{\left(t,\frac{1}{t}\right)} = -\frac{1}{t^2}.$$

$\Rightarrow$ Slope of the normal at $\left(t, \frac{1}{t}\right)$ is t^2.

Since ax + by + c = 0 is normal to xy = 1, therefore slope of the normal is $-\frac{b}{a}$.

Thus, $t^2 = -\frac{b}{a}$

$\Rightarrow$ b and a are of opposite sings

$\Rightarrow$ either b > 0 and a < 0 or b < 0 and a > 0

Sol.5 (A)

The equations of the tangents at the origin can be obtained by equating the lowest degree term to zero i.e.

$y^2 - x^2 = 0 \Rightarrow y = \pm x$

Sol.6 (B)

We have, $\sqrt{x} + \sqrt{y} = 3$.

$$\Rightarrow \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}} \qquad \Rightarrow \left(\frac{dy}{dx}\right)_{(4,1)} = -2$$

$$\therefore \text{ Length of the subtangent} = \left|\frac{y}{dy/dx}\right| = \frac{1}{2}$$

Sol.7 (B)

By lagrange's mean value theorem there exists $c \in (1, 6)$ such that

$$f'(c) = \frac{f(6) - f(1)}{6 - 1}$$

$$\Rightarrow \frac{f(6) + 2}{5} = f'(c) \geq 2$$

$$[\because f'(x) \geq 2 \text{ for all } x \in [1, 6]]$$

$$\Rightarrow f(6) + 2 \geq 10 \qquad \Rightarrow f(6) \geq 8$$

Sol.8 (C)

$f(x) = \cos|x| - 2ax + b$

$= \cos x - 2ax + b \; [\because \cos(-x) = \cos x]$

$\Rightarrow f'(x) = -\sin x - 2a$

Now, f(x) is increasing for all $x \in R$

$\Rightarrow f'(x) \geq 0$ for all $x \in R$

$$\Rightarrow f'\left(\frac{\pi}{2}\right) \geq 0 \quad \Rightarrow \quad -\sin\frac{\pi}{2} - 2a \geq 0$$

$$\Rightarrow -1 - 2a \geq 0 \Rightarrow a \leq -1/2$$

Sol.9 (B)

Since $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing for all real values of x, therefore

$f'(x) < 0$ for all x

$$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0 \text{ for all } x$$

$$\Rightarrow \sqrt{3}\cos x + \sin x < 2a \text{ for all } x$$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a \text{ for all } x$$

$$\Rightarrow \cos x \sin\frac{\pi}{3} + \sin x \cos\frac{\pi}{3} < a \text{ for all } x$$

$$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a \text{ for all } x$$

$$\Rightarrow a \geq 1$$

Sol.10 (B)

We have, $f(x) = e^{ax} + e^{-ax}$.

$\Rightarrow f'(x) = a[e^{ax} - e^{-ax}]$

$$= 2a\left[ax + \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!} + \ldots\right]$$

$$= 2a^2x\left[1 + \frac{a^2x^2}{3!} + \frac{a^4x^4}{5!} + \ldots\right]$$

Now,

$f'(x) < 0 \;\Rightarrow\; x < 0$

$$\left[\because 2a^2\left\{1 + \frac{a^2x^2}{3!} + \frac{a^4x^4}{5!} + \ldots\right\} > 0\right]$$

Hence, f(x) is decreasing for $x < 0$.