Dear student following is a Moderate level [○●○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,−1). (Questions may have more than one option correct).

Q.1 Let f(x) be a monotonic polynomial of 2m − 1 degree where $m \in N$ then the equation $f(x) + f(3x) + f(5x) + ... f(2m-1)x = 2m-1$ has
(A) atleast one real root
(B) (2m − 1) real roots
(C) exactly one real root
(D) None of these

Q.2 The function $f(x) = |ax - b| + c|x| \ \forall x \in (-\infty, \infty)$, where a > 0, b > 0, c > 0, assume it's minimum only at one point if
(A) $a \neq b$ (B) $a \neq c$
(C) $b \neq c$ (D) a = b = c

Q.3 If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal roots, then-
(A) $2b^2 - 10a^3b^2 + c^5 = 0$
(B) $6a^5 + c^5 = 0$
(C) $2c^5 - 10a^3b^2 + b^4c^5 = 0$
(D) $b^4 = 15a^4$

Q.4 Let S be the set of real values of parameter λ for which the function $f(x) = 2x^3 - 3(2+\lambda)x^2 + 12\lambda x$ has exactly one local maximum exactly one local minimum, then subsets of S are-
(A) $(-4, \infty)$ (B) (−3, 3)
(C) $(3, \infty)$ (D) $(-\infty, 0)$

Q.5 If the function y = sin (f(x)) is monotonic for all values of x (where f(x) is continuous), then the maximum value of the difference between the maximum and the minimum value of f(x), is-
(A) π (B) 2π
(C) $\frac{\pi}{2}$ (D) None of these

Q.6 The maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2 + (x-5)^2$, (where $1 \le x \le 3$) is :
(A) 34 (B) 36 (C) 32 (D) 20

Q.7 A solid cylinder of height H has a conical portion of same height and radius 1/3rd of height removed from it. Rain water is falling in the cylinder with rate equal to π times the instantaneous radius of the water surface inside hole, the time after which hole will fill up with water is :
(A) $\frac{H^2}{3}$ (B) H^2 (C) $\frac{H^2}{6}$ (D) $\frac{H^2}{4}$

Q.8 If the relation between subnormal SN and subtangent ST at any point S on the curve; $by^2 = (x+a)^3$ is $p(SN) = q(ST)^2$, then find the value of $\frac{p}{q}$.
(A) $\frac{8b}{27}$ (B) $\frac{27b}{8}$ (C) $\frac{8a}{27}$ (D) $\frac{27a}{8}$

Q.9 If g(x) is a curve which is obtained by the reflection of $f(x) = \frac{e^x - e^{-x}}{2}$ by the line y = x then :
(A) g(x) has more than one tangent parallel to x-axis
(B) g(x) has more than one tangent parallel to y-axis
(C) y = − x is a tangent to g(x) at (0, 0)
(D) g(x) has no extremum.

MATHEMATICS IIT JEE (JULY 4th WEEK CLASS TEST 4) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○					
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	C	B	B,D	C,D	A	B	C	A	D

SOLUTIONS

Sol.1 (C)

f(x) is monotonic

$\Rightarrow$ $f'(x) < 0$ or $f'(x) > 0$ $\forall x \in R$

$\Rightarrow$ $f'(px) < 0$ or $f'(px) > 0$ $\forall x \in R$

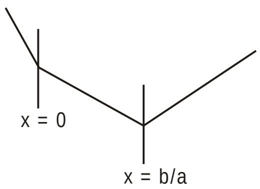
$\Rightarrow$ f(px) is also monotonic

$\Rightarrow$ $f(x) + f(3x) + \ldots. f(2m - 1)x$ is a monotonic polynomial of odd degree (2m – 1), so it will attain all real values only once.

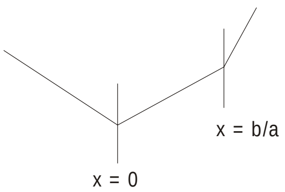
Sol.2 (B)

$$f(x) = \begin{cases} b-(a+c)x & , \quad x<0 \\ b+(c-a)x & , \quad 0 \le x < \dfrac{b}{a} \\ (a+c)x+b & , \quad x \ge \dfrac{b}{a} \end{cases}$$

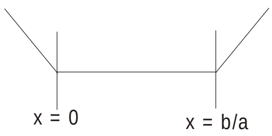
Case I : When c < a



Case II : When c > a



Case III : when c = a



Sol.3 (B, D)

$f(x) = x^5 - 10a^3x^2 + b^4x + c^5$

$f'(x) = 5x^4 - 20a^3x + b^4$

$f'(x) = 20x^3 - 20a^3$

If $x = \alpha$ be a root that is repeated three times

$\Rightarrow$ $f''(\alpha) = 0, f'(\alpha) = 0, f(\alpha) = 0$

$\Rightarrow$ $\alpha = a$, $5a^4 - 20a^4 + b^4 = 0$

$a^5 - 10a^5 + ab^4 + c^5 = 0$

$\Rightarrow$ $\alpha = a$, $b^4 = 15a^4$ and $c^5 + 15a^5 - 9a^5 = 0$

or $6a^5 + c^5 = 0$

Sol.4 (C, D)

$f(x) = 2x^3 - 3(2 + \lambda)x^2 + 12\lambda x$

$f'(x) = 6x^2 - 6(2 + \lambda)x + 12\lambda$

$f'(x) = 0 \Rightarrow x = 2, \lambda$, then $\lambda \neq 2$.

Sol.5 (A)

As, y = sin (f(x)) is monotonic for

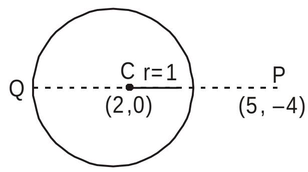
$$f(x) \in \left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right]$$

or $\left[2n\pi + \dfrac{\pi}{2}, 2n\pi + \dfrac{3\pi}{2}\right]$

$\therefore$ The maximum value of difference is π.

Sol.6 (B)

Here, $\left(\sqrt{-3+4x-x^2}+4\right)^2 + (x - 5)^2$, represents the square of the distance between circle $y = \sqrt{-3+4x-x^2}$ and point (5, –4).



i.e., maximum distance between $x^2 + y^2 - 4x + 3 = 0$ and (5, – 4) square

$\Rightarrow$ $PQ^2 = (PC + \text{radius})^2$

$$= \left(\sqrt{(5-2)^2+(4-0)^2}+1\right)^2$$

$= 6^2 = 36$.

Sol.7 (C)

Here, $r = \dfrac{H}{3}$, $\dfrac{x}{r} = \dfrac{y}{H}$

$\Rightarrow$ $3x = y$

$$\frac{dv}{dt} = \pi x$$

$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{3}\pi x^2 y\right) = \pi x$

$\Rightarrow \quad 3\int_0^r x\,dx = \int_0^t dt$

$\Rightarrow \quad 3\frac{r^2}{2} = t \Rightarrow \frac{3}{2}\cdot\frac{H^2}{9} = t$

$\Rightarrow \quad t = \frac{H^2}{6}$

Sol.8 (A)

Here, $by^2 = (x + a)^3$,

Differentiating both sides, we get

$$2by\frac{dy}{dx} = 3(x + a)^2 \cdot 1$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3}{2}\frac{(x+a)^2}{by}$

$\therefore$ Length of subnormal

$\Rightarrow \quad SN = y\frac{dy}{dx} = \frac{3}{2}\frac{(x+a)^2}{b}$ .... (1)

and length of subtangent

$\Rightarrow \quad ST = y\frac{dx}{dy} = \frac{2by^2}{3(x+a)^2}$ .... (2)

$\therefore \quad \frac{p}{q} = \frac{(ST)^2}{(SN)}$ (given)

$\Rightarrow \quad \frac{p}{q} = \frac{(2by^2)^2.2b}{\{3(x+a)^2\}^2.3(x+a)^2}$

{using, (1) and (2)}

$= \frac{8b\{(x+a)^3\}^2}{27(x+a)^6}$

{using, $by^2 = (x + a)^3$}

$= \frac{8b}{27}$

$\therefore \quad \frac{p}{q} = \frac{8b}{27}$

Sol.9 (D)

As g(x) is a curve which is obtained by the reflection of $f(x) = \frac{e^x - e^{-x}}{2}$ on $y = x$

$\Rightarrow$ g(x) is inverse of f(x)

$\therefore \quad g(x) = \log\left(x + \sqrt{1+x^2}\right) = f^{-1}(x)$

$\Rightarrow \quad g'(x) = \frac{1}{x+\sqrt{1+x^2}}\cdot\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)$

$= \frac{1}{\sqrt{1+x^2}} \neq 0, \ \forall\ x \in R$

$\Rightarrow$ g(x) has no tangent parallel to x-axis also g'(x) is always defined, $\forall\ x \in R$.

$\Rightarrow$ g(x) has no tangent parallel to y-axis since, $g'(x) > 0$

$\Rightarrow$ g(x) has not any extremum.