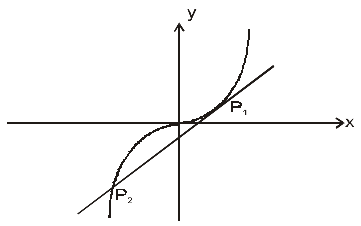
Dear student following is an Easy level [● ○ ○] test paper. Score of 24 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3, –1) (All questions have only one option correct)

PASSAGE - 1



Tangent at a point P_1 (x_1, y_1) (other than origin) on the curve $y = x^3$ drawn and meets again the curve at P_2. The tangent at P_2 meets the curve at P_3, and so, on Hence we get points P_1, P_2, P_3...... P_n and they are joined to each other to form triangles

Q.1 What is the location of origin wrt the triangle $P_i\ P_{i+1}\ P_{i+2}$ for any i = 1,2,3.....
(A) Inside the triangle
(B) Outside
(C) On the triangle
(D) Can't predict

Q.2 The ordinates of points P_1, P_2, P_3.....P_n form a sequence of
(A) AP (B) GP (C) HP (D) None

Q.3 Ratio $\dfrac{\text{area}(\Delta P_i P_{i+1} P_{i+2})}{\text{area}(\Delta P_{i+1} P_{i+2} P_{i+3})} =$

(A) $\frac{1}{12}$ (B) $\frac{1}{16}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

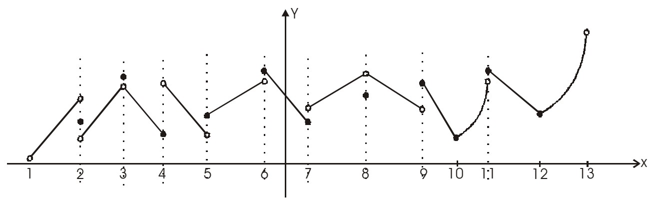
Q.4 If area of $(\Delta\ P_1P_2P_3)$ = A then value of

$$\sum_{i=1}^{\infty}\left[\frac{1}{\text{area}(\Delta P_i P_{i+1} P_{i+2})}\right] =$$

(A) ∞ (B) $\frac{16}{15A}$ (C) $\frac{15}{16A}$ (D) $\frac{1}{16A}$

PASSAGE - 2

Let a function f : D $\to$ R, is defined as.



Concept of maxima-minima is to be checked here.

Q.5 Point 1 is the point of
(A) Maxima (B) Minima
(C) Neither maxima nor minima
(D) Can't say

Q.6 Point 3 is the point of
(A) Maxima (B) Minima
(C) Neither maxima nor minima
(D) Can't say

Q.7 Point 4 is the point of
(A) Maxima (B) Minima
(C) Neither maxima nor minima
(D) Can't say

Q.8 Point 5 is the point of
(A) Maxima (B) Minima
(C) Neither maxima nor minima
(D) Can't say

Q.9 Point 10 is the point of
(A) Maxima (B) Minima
(C) Neither maxima nor minima
(D) Can't say

Q.10 At which point, y = f(x) has global maxima or minima ?
(A) Function does not have global maxima or minima
(B) 1 (C) 13
(D) $x \to 13^-$ and $x \to 1^+$

MATHEMATICS IIT JEE (AUGUST 4th WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	B	B	D	A	B	C	B	A

SOLUTIONS

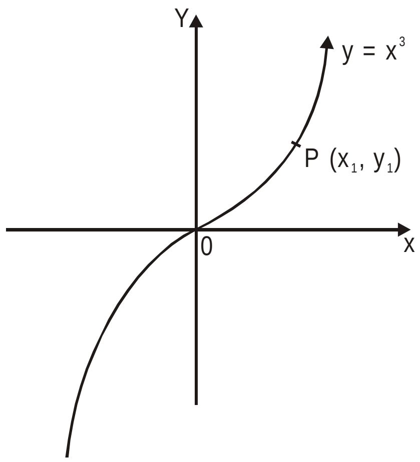
PASSAGE - 1

Sol.1 (A) **Sol.2** (B) **Sol.3** (B) **Sol.4** (B)

Let $P_1(x_1, y_1)$ be a point on the curve

$y = x^3$ ...(i)

$y_1 = x_1^3$ ...(ii)



Now, $\frac{dy}{dx} = 3x^2$

$\therefore$ Slope of the tangent at $P_1 = m_1 = 3x_1^2$

$\therefore$ Equation of the tangent at $P_1(x_1, y_1)$ is

$y - x_1^3 = 3x_1^2 (x - x_1)$

i.e., $y = 3x_1^2 x - 2x_1^3$ ...(iii)

Solving (i) and (iii), we get

$x^3 - 3x_1^2 x + 2x_1^3 = 0$

i.e., $(x - x_1).(x^2 + xx_1 - 2x_1^2) = 0$

i.e., $(x - x_1)(x - x_1)(x + 2x_1) = 0$

$\therefore$ $x = x_1$ (neglecting) or $x = -2x_1$

$\therefore$ $x_2 = -2x_1$, $y_2 = x_2^3 = -8x_1^3$

$\therefore$ $P_2(x_2, y_2) = (-2x_1, -8x_1^3)$

Now, we find P_3, the point where the curve meets again at P_3.

Slope of the tangent at

$$P_2 = \left(\frac{dy}{dx}\right)_{(x_2, y_2)}$$

$$= 3x_2^2 = 3.4x_1^2 = 12x_1^2$$

$\therefore$ Equation of tangent at P_2 is,

$y - x_2^3 = 3x_2^2(x - x_2)$ ..(iv)

To get $P_3 = (x_3, y_3)$, solve (i) and (iv)

$\therefore$ $P_3 = (x_3, y_3) = (-2x_2, -8x_2^3)$

$= (4x_1, 64x_1^3)$ and so on.

$\therefore$ Abscissa of P_1, P_2, P_3,.... are given by $x_1, -2x_1, 4x_1, -8x_1$,..., which is GP with common ratio = - 2

Now, area $(\Delta P_1P_2P_3) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

area $(\Delta P_1P_2P_3) = \frac{1}{2}\begin{vmatrix} x_1 & x_1^3 & 1 \\ -2x_1 & -8x_1^3 & 1 \\ 4x_1 & 64x_1^3 & 1 \end{vmatrix}$

area $(\Delta P_1P_2P_3) = \frac{x_1^4}{2}\begin{vmatrix} 1 & 1 & 1 \\ -2 & -8 & 1 \\ 4 & 64 & 1 \end{vmatrix}$

Similarly,

area $(\Delta P_2P_3P_4) = \frac{1}{2}\begin{vmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$

$$\text{area } (\Delta\ P_2P_3P_4) = \frac{1}{2}\begin{vmatrix} -2x_1 & -8x_1^3 & 1 \\ 4x_1 & 64x_1^3 & 1 \\ -8x_1 & 512x_1^3 & 1 \end{vmatrix}$$

$$\text{area } (\Delta\ P_2P_3P_4) = 8x_1^4\begin{vmatrix} 1 & 1 & 1 \\ -2 & -8 & 1 \\ 4 & 64 & 1 \end{vmatrix}$$

$$\therefore \quad \frac{\text{area}\,(\Delta P_1P_2P_3)}{\text{area}\,(\Delta P_2P_3P_4)} = \frac{1}{16}$$

PASSAGE - 2

Sol.5 (D)

Sol.6 (A)

Sol.7 (B)

Sol.8 (C)

Sol.9 (B)

Sol.10 (A)