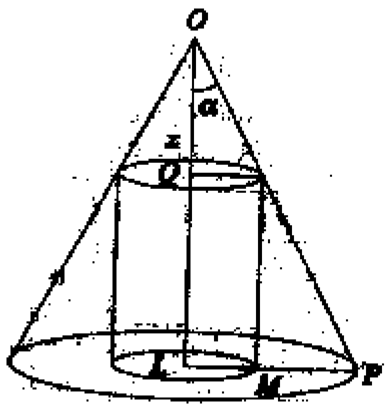
Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3, –1) (All questions have only one option correct)

Q.1 Three normals are drawn to the parabola $y^2 = 4x$ from the point (c, 0). These normals are real and distinct when-

(A) c = 0 (B) c = 1
(C) c = 2 (D) c = 3

Q.2 A given right circular cone has a volume p, and the largest right circular cylinder that can be inscribed in the cone has a volume q. Then p : q is-



(A) 9 : 4 (B) 8 : 3
(C) 7 : 2 (D) None of these

Q.3 The sum of the intercepts of a tangent to $\sqrt{x} + \sqrt{y} = \sqrt{a}$, a > 0 upon the coordinate axes is-

(A) 2a (B) a (C) a/2 (D) $\sqrt{a}$

Q.4 If $f(x) = x^{2/3}$ then-

(A) (0, 0) is a point of maximum
(B) (0, 0) is not a point of minimum
(C) (0, 0) is a critical point
(D) There is no critical point

Q.5 An equation of the circle that is tangent to $y = x^3$ at (1, 1) and has the same second derivative there is-

(A) $x^2 + y^2 + 24x - 28y + 2 = 0$
(B) $2(x^2 + y^2) + 12x - 8y - 8 = 0$
(C) $3(x^2 + y^2) - 24x + 10y + 8 = 0$
(D) None of these

Q.6 The maximum area of the rectangle whose sides pass through the angular points of a given the rectangle is of sides a and b is-

(A) $(1/2)\,(ab)^2$ (B) (1/2) (a + b)
(C) $(1/2)\,(a + b)^2$ (D) None of these

Q.7 The equation $e^{x-8} + 2x - 17 = 0$ has

(A) Two real roots (B) One real root
(C) Eight real roots (D) Four real roots

Q.8 A differentiable function f(x) has a relative minima at x = 0 then the function y = f(x) + ax + b has a relative minima at x = 0 for-

(A) All a and all b (B) All b if a = 0
(C) All b > 0 (D) All a > 0

Q.9 The graph of the function y = f(x) has a unique tangent at the point (a, 0) through which it passes. Then $\lim_{x \to a} \dfrac{\log_e\{1 + 6f(x)\}}{3f(x)}$ equals-

(A) 1 (B) 0
(C) 2 (D) None of these

Q.10 Let $f(x) = (x - 1)^4 (x - 2)^n$ $n \in N$. Then f(x) has a-

(A) Max. at x = 1 if n is odd
(B) Max. at x = 1 if n is even
(C) Neither max. nor minimum at x = 1 for odd n
(D) None of these

MATHEMATICS IIT JEE (AUGUST 4th WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
										10	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	B	C	D	C	B	B	C	A

SOLUTIONS

Sol.1 (D)

Any point on $y^2 = 4x$ is $(t^2, 2t)$.

Since $\frac{dy}{dx} = \frac{2}{y}$ so $\left.\frac{dy}{dx}\right|_{(t^2,\,2t)} = \frac{2}{2t} = \frac{1}{t}$.

Hence equation of normal at (t^2, t) is

$Y - 2t = -t(X - t^2)$

This passes through (c, 0)

if $-2t = -t(c - t^2)$

$\Rightarrow$ $t = 0,\ t^2 = c - 2$

Thus the roots are real and distinct if $c > 2$ so $c = 3$ is the correct choice.

Sol.2 (A)

Let H be the height of the cone and α be its semi vertical angle. Suppose that x is the radius of the inscribed cylinder and h be its height $h = QL = OL - OQ = H - x\cot\alpha$

V = volume of the cylinder

$= \pi x^2 (H - x\cot\alpha)$

Also $p = \frac{1}{3}\pi(H\tan\alpha)^2 H$ ....(1)

$$\frac{dV}{dx} = \pi(2Hx - 3x^2\cot\alpha)$$

So $\frac{dV}{dx} = 0 \Leftrightarrow x = 0,\ x = \frac{2}{3}H\tan\alpha$,

$$\left.\frac{d^2V}{dx^2}\right|_{x=\frac{2}{3}H\tan\alpha} = -2\pi H < 0.$$

So V is maximum when $x = 2/3H\tan\alpha$ and

$q = V_{max} = \pi\frac{4}{9}H^2\tan^2\alpha\,\frac{1}{3}H$

$= \frac{4}{9}p$. [using (1)]

Hence $p : q = 9 : 4$

Sol.3 (B)

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}.$$

Equation of tangent at any point (x, y) of the curve is $Y - y = -\frac{\sqrt{y}}{\sqrt{x}}(X - x)$. So intercepts of X-axis and Y-axis are $x + \sqrt{xy}$ and $y + \sqrt{xy}$

So the sum of intercepts $= x + y + 2\sqrt{xy}$

$$= \left(\sqrt{x} + \sqrt{y}\right)^2 = a.$$

Sol.4 (C)

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3}$. This derivative is never zero, but there is no derivative for $x = 0$. So (0, 0) is a critical point. If $x < 0$ then $\frac{dy}{dx} < 0$ and if $x > 0$ then $\frac{dy}{dx} > 0$. Thus (0, 0) is a point of minimum.

Sol.5 (D)

$\frac{dy}{dx} = 3x^2$ so $\left.\frac{dy}{dx}\right|_{(1,1)} = 3$ and $\left.\frac{d^2y}{dx^2}\right|_{(1,1)} = 6$

Let the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.

Since this should pass through (1, 1) so we have

$2 + 2g + 2f + c = 0$ ....(1)

Also $2x + 2y\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$

Putting $x = 1$, $y = 1$ and $\frac{dy}{dx} = 3$, we have

$1 + 3 + g + 3f = 0$

Again differentiating (1), we have

$$1 + \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} + f\frac{d^2y}{dx^2} = 0$$

Putting $y = 1$, $\frac{dy}{dx} = 3$ and $\frac{d^2y}{dx^2} = 6$, we have

$16 + 6f = 0 \Rightarrow f = -8/3$

so $g = 4$ and $c = -14/3$.

Thus required circle is

$3(x^2 + y^2) + 24x - 16y - 14 = 0.$

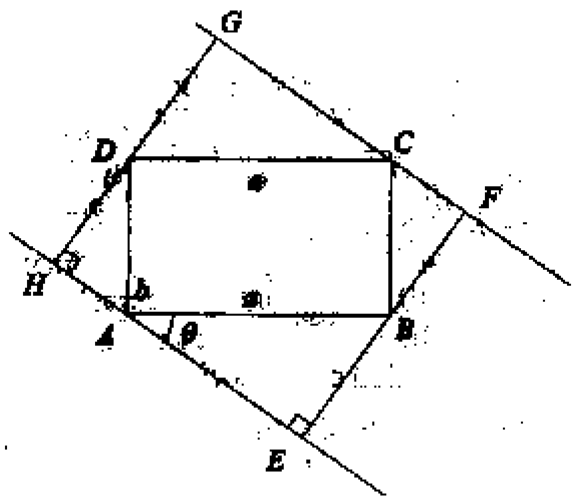
Sol.6 (C)

Let ABCD be the given rectangle of sides a and b and EFGH be any rectangle, whose sides pass through A, B, C, D

A = Area EFGH = $(b \sin\theta + a\cos\theta)$

$(a\sin\theta + b\cos\theta)$

$= ab + (a^2 + b^2)\sin\theta\cos\theta$



$dA/d\theta = (a^2 + b^2)\cos 2\theta$ so $dA/d\theta = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

$$\Rightarrow \frac{d^2A}{d\theta^2} = 1 - 2(a^2 + b^2)\sin 2\theta,$$

so $\left.\frac{d^2A}{d\theta^2}\right|_{\theta=\pi/4} < 0$

Hence $A_{max} = (1/2)(a + b)^2$.

Sol.7 (B)

Clearly $x = 8$ satisfies the given equation. Assume that $f(x) = e^{x-8} + 2x - 17 = 0$ has a real root α other than $x = 8$. We may suppose that $\alpha > 8$ (the case for $\alpha < 8$ is exactly similar). Applying Rolle's theorem on $[8, \alpha]$, we get $\beta \in (8, \alpha)$, such that $f'(\beta) = 0$. But $f'(\beta) = e^{\beta-8} + 2$, so that $e^{\beta-8} = -2$ which is not possible. Hence there is no real root other than 8.

Sol.8 (B)

Given that $y = f(x) + ax + b$ .....(1)

Since $f(x)$ has minima at $x = 0$

$\Rightarrow f'(0) = 0$ and $f''(0) < 0$

Further $y' = f'(x) + a$

and $(y'')_0 = f''(0) < 0$

Now y will also have a minima at $x = 0$ if

$(y')_{x=0} = 0 \Rightarrow f'(0) + a = 0$

$\Rightarrow a = 0$

Sol.9 (C)

$y = f(x)$ passes through $(a, 0)$

$\Rightarrow 0 = f(a)$ .... (1)

It has a unique tangent at(a, 0) means $f'(a)$ exists.

Then $\lim_{x\to a}\frac{\log(1+6f(x))}{3f(x)}$ $\left(\because \frac{\log 1}{0} = \frac{0}{0}\right)$

$$= \lim_{x\to a}\frac{\frac{1}{1+6f(x)}.6f'(x)}{3f'(x)} \text{ By L.H. Rule}$$

$$= \lim_{x\to a}\frac{2}{1+6f(x)} = \frac{2}{1+6f(a)} = 2$$

Sol.10 (A)

$f(x) = (x - 1)^4 (x - 2)^n$

$\Rightarrow \quad f'(x) = 4(x - 1)^3 (x - 2)^n + n(x - 1)^4 (x - 2)^{n-1}$

$= (x - 1)^3 (x - 2)^{n-1} [4(x - 2) + n(x - 1)]$

$f'(x) = (x - 1)^3 (x - 2)^{n-1} [(4 + n)x - (8 + n)]$

Then $f'(x) = 0 \Rightarrow x = 1, 2, \dfrac{8+n}{4+n}$

At $x = 1 - h$ we have

$f'(1 - h) = -h^3(-1 - h)^{n-1} [(4 + n)(1 - h) - (8 + n)]$

$f'(1 - h) = (-1)^{n+1} (1 + h)^{n-1} h^3 [4 + (4 + n)h]$

And $f'(1 + h) = h^3(-1)^{n-1} (1 - h)^{n-1} [(4 + n)(1 + h) - (8 + n)]$

$f'(1 + h) = (-1)^n h^3 (1 - h)^{n-1} [4 - (4 + n)h]$

We observe that if n is odd and h is small then

$f'(1 - h) > 0$ and $f'(1 + h) < 0$

Thus f(x) will have a maxima at x = 1 if n is odd.