Dear student following is a Moderate level [○●○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Find the equation of familyof curves for which the length of normal is equal to the radius vector.

(A) $y^2 = \pm x^2 - c$ (B) $y^2 = \pm x^2 + c$

(C) $y^2 = x + c$ (D) None

Q.2 The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at the point whose abscissa is 0 is

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{3}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{3}}$

Q.3 Let f(x) and g(x) be defined and differentiable for $x \geq x_0$ and $f(x_0) = g(x_0)$, $f'(x) > g'(x)$ for $x > x_0$ then

(A) $f(x) < g(x)$ for some $x > x_0$

(B) $f(x) = g(x)$ for some $x > x_0$

(C) $f(x) > g(x)$ for all $x > x_0$

(D) None of these

Q.4 The function $f(x) = \sum_{k=1}^{5} (x-k)^2$ assumes minimum value for x given by

(A) 3 (B) 2 (C) $\frac{5}{2}$ (D) 5

Q.5 For the curve $y^n = a^{n-1} x$ if the subnormal at any point is a constant then n is equal to

(A) 1 (B) 2 (C) –2 (D) –1

Q.6 Let $f(x) = 1 + 3x^2 + 3^2x^4 + ... + 3^{30}.x^{60}$. Then f(x) has

(A) atleast one maximum

(B) exactly one maximum

(C) atleast one minimum

(D) exactly one minimum

Q.7 The equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at x = 0 is

(A) x + y = 2 (B) x + y = 1

(C) x – y = 1 (D) None of these

Q.8 The critical points of the function f'(x), where $f(x) = \frac{|x-2|}{x^3}$ are

(A) 0 (B) 1 (C) 3 (D) –1

Q.9 The function $f(x) = \frac{\log(\pi + x)}{\log(e + x)}$ is

(A) increasing on $[0, \infty)$

(B) decreasing on $[0, \infty)$

(C) increasing on $[0, \pi/e)$ and decreasing on $[\pi/e, \infty)$

(D) decreasing on $[0, \pi/e)$ and increasing on $[\pi/e, \infty)$

Q.10 An extremum value of the function $f(x) = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$ $(-1 < x < 1)$ is

(A) $\frac{7\pi^3}{8}$ (B) $\frac{\pi^3}{8}$ (C) $\frac{\pi^3}{32}$ (D) $\frac{\pi^3}{16}$

MATHEMATICS IIT JEE (JULY 5th WEEK CLASS TEST 1) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

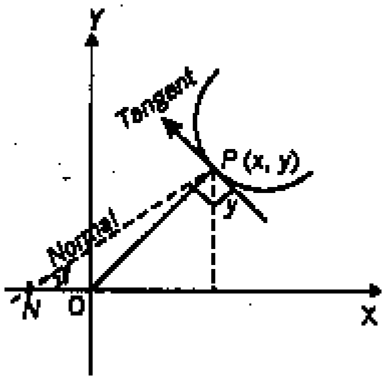
ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	C	A	B	D	B	C	B	C

SOLUTIONS

Sol.1 (B)

Let P(x, y) be the point on the curve.



OP = radius vector = $\sqrt{x^2 + y^2}$

PN = length of normal

Now, $\tan\phi = -\dfrac{1}{\left(\dfrac{dy}{dx}\right)}$

$\Rightarrow \quad PN = \dfrac{y}{\sin\phi}$

It is given OP = PN

$\Rightarrow \quad \sqrt{x^2 + y^2} = y\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

$\Rightarrow \quad x^2 + y^2 = y^2\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]$

$\Rightarrow \quad x^2 = y^2\left(\dfrac{dy}{dx}\right)^2$

$\Rightarrow \quad \dfrac{dy}{dx} = \pm\dfrac{x}{y}$

or $\quad y\,dy = \pm x\,dx$

integrating both sides,

$y^2 = \pm x^2 + c$ is required family of curves.

Sol.2 (B)

The point on the curve corresponding to x = 0 is (0, 1)

$\dfrac{dy}{dx} = 2e^{2x} + 2x$

$\Rightarrow \quad \left.\dfrac{dy}{dx}\right|_{x=0} = 2$

Hence the equation of the normal at the point (0, 1) is

$y - 1 = \left(-\dfrac{1}{2}\right)(x - 0)$

$\Rightarrow \quad 2y + x - 2 = 0$

Therefore, the distance of the point (0, 0) from this line is $\dfrac{2}{\sqrt{5}}$.

Sol.3 (C)

Consider the function $\phi(x) = f(x) - g(x)$ on the interval $[x_0, x]$.

By Lagrange's theorem we have

$\phi(x) - \phi(x_0) = \phi'(z)(x - x_0)$

for some $z \in (x_0, x)$.

Since $\phi(x_0) = 0$, $\phi'(z) = f'(z) - g'(z) > 0$.

So $\phi(x) - \phi(x_0) = \phi(x)$

$= (f'(z) - g'(z))(x - x_0) > 0$

$\Rightarrow \quad \phi(x) > 0$

Thus $f(x) > g(x)$ for all $x > x_0$.

Sol.4 (A)

We have

$f(x) = (x - 1)^2 + (x - 2)^2 + (x - 3)^2 + (x - 4)^2 + (x - 5)^2$

$\Rightarrow \quad f'(x) = 2[x - 1 + x - 2 + x - 3 + x - 4 + x - 5]$

$= 2(5x - 15)$.

$f'(x) = 0$

$\Rightarrow \quad 2(5x - 15) = 0$

$\Rightarrow \quad x = 3$

Also, $f''(x) = 10 > 0$, for all x.

$\therefore \quad f(x)$ is minimum for x = 3.

Sol.5 (B)

For the curve $y^n = a^{n-1}x$

On putting n = 2, we get

$y^2 = ax$

which is curve of the form of parabola, where the subnormal at any point is a constant.

$\therefore \quad n = 2$.

Sol.6 (D)

We have,

$f(x) = 1 + 3x^2 + 3^2.x^4 + ... 3^{30}.x^{60}$

$\Rightarrow$ $f'(x) = x(6 + 4.3^2.x^2 + ... 60.3^{30}.x^{58})$

$f'(x) = 0$

$\Rightarrow$ $x = 0.$

Also, $f''(x) = 1.(6 + 4.3^2x^2 + ... 60.3^{30}.x^{58})$

$+ x(4.3^2x^2 . 2x + ... 60.3^{30}.58.x^{57})$

$\Rightarrow$ $f''(0) = 6 > 0.$

$\therefore$ f(x) has minimum at x = 0 only.

Sol.7 (B)

We have, $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$

$$\Rightarrow \frac{dy}{dx} = (1 + x)^y \left(\frac{y}{1+x} + \frac{dy}{dx}\log(1+x)\right) + \frac{2\sin x\cos x}{(1-\sin^4 x)^{1/2}}$$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{x=0} = 1.\left(\frac{y(0)}{1} + \left.\frac{dy}{dx}\right|_{x=0}\log 1\right) + 0$$

$= y(0) = 1.$

$\therefore$ The equation of the normal at (0, 1) is $(y - 1) = -1\,(x - 0)$ i.e. $x + y = 1$.

Sol.8 (C)

$$f(x) = \begin{cases} \dfrac{x-2}{x^3}, & x > 1 \\ \dfrac{2-x}{x^3}, & x < 1,\ x \neq 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{2(3-x)}{x^4}, & x \in (-\infty, 0) \cup (0, 1) \\ \dfrac{2(x-3)}{x^4}, & x \in (1, \infty) \end{cases}$$

$$f''(x) = \begin{cases} \dfrac{6(x-4)}{x^5}, & x \in (-\infty, 0) \cup (0, 1) \\ \dfrac{6(4-x)}{x^5}, & x \in (1, 3) \cup (3, \infty) \end{cases}$$

f''(x) doesn't exist at x = 3. Thus critical point of f'(x) is 3.

Sol.9 (B)

$$f'(x) = \frac{\log(e+x)\times\dfrac{1}{\pi+x} - \log(\pi+x)\times\dfrac{1}{e+x}}{(\log(e+x))^2}$$

Since $\pi + x > e + x$ for $x > 0$, and log x is an increasing function so

$\log(\pi + x) > \log(e + x).$

Thus

$(\pi + x)\log(\pi + x).(\pi + x)\log(e + x)$

$> (e + x)\log(e + x).$

Hence $f'(x) < 0$, for $x > 0$.

Thus f decreases on $[0, \infty)$.

Sol.10 (C)

$f'(x) = 3[(\sin^{-1} x)^2 - (\cos^{-1} x)^2] \,/\, \left(\sqrt{1-x^2}\right).$

For extremum value, we have $f'(x) = 0$

so $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = 0$

i.e. $\sin^{-1} x - \cos^{-1} x = 0$, or $\sin^{-1} x + \cos^{-1} x = 0$ but since $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ so the equality is not possible. Hence $x = \sqrt{2}$, thus

$$f\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)^3 + \left(\frac{\pi}{4}\right)^3 = \frac{\pi^2}{32}.$$