Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-8(+3,–1), 9(+6, 0). (All questions have only one option correct).

PASSAGE I (Q.NO. 1 - 2) :

Let f(x) be a continuous, differentiable and bijective function. If the tangent to $y = f(x)$ at $x = a$, is also the normal to $y = f(x)$ at $x = b$. Then

Q.1 We must have atleast one value c belonging to (for which $f'(c) = 0$)

(A) $\left(\frac{a}{2}, b\right)$ (B) $\left(a, \frac{b}{2}\right)$

(C) (0, a) (D) None of these

Q.2 At the point $x = c$, we will have a point of

(A) Local maxima (B) Local minima

(C) Nothing can be said (D) None of these

PASSAGE II (Q.NO. 3 - 5) :

If $f : R \to R$, and f(x) is a differentiable function such that all its successive derivatives exists. $f'(x)$ can be zero at discrete points only and $f(x) \cdot f''(x) \le 0$, $\forall\ x \in R$, then.

Q.3 If $f(a) = 0$, then which of the following is correct

(A) $f(a + h) \cdot f''(a - h) < 0$

(B) $f(a + h) f''(a - h) > 0$

(C) $f'(a + h) f''(a - h) < 0$

(D) $f'(a + h) \cdot f''(a - h) < 0$

Q.4 If α and β are two consecutive roots of $f(x) = 0$, then

(A) $f''(r) = 0, r \in (\alpha, \beta)$

(B) $f'''(r) = 0, r \in (\alpha, \beta)$

(C) $f''''(r) = 0, r \in (\alpha, \beta)$

(D) $f'''''(r) = 0, r \in (\alpha, \beta)$

Q.5 If $f'(x) \neq 0$, the maximum number of real roots of $f''(x) = 0$ is/are :

(A) No real root (B) One

(C) Two (D) Three

Q.6 If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c is-

(A) No relation (B) $0 < c < b/2$

(C) $|c| < |b|\sqrt{2}$ (D) $|c| > |b|\sqrt{2}$

Q.7 Let $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$, then at $x = 0$, f has-

(A) A local maximum (B) No local maximum

(C) A local minimum (D) No extremum

Q.8 If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number, then minimum value of f-

(A) Does not exist

(B) Is not attained even though f is bounded

(C) Is equal to 1

(D) Is equal to – 1

Q.9 Match the difference of greatest and least value of the functions f in column 1

(i) $\sin x \sin 2x$ on $(-\infty, \infty)$ (a) $\frac{\pi}{6}$

(ii) $2x^3 - 3x^2 - 12x + 1$ on $[-2, 5/2]$ (b) $\frac{8}{3\sqrt{3}}$

(iii) $\cos^{-1} x^2$ on $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$ (c) 6

(iv) $x + \sqrt{x}$ on [0, 4] (d) 27

MATHEMATICS IIT JEE (JULY 5th WEEK CLASS TEST 2) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		a	b	c	d
1	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	9	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭	(i)	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭	(ii)	⬭	⬭	⬭	⬭
4	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭	(iii)	⬭	⬭	⬭	⬭
										(iv)	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9			
Ans.	D	D	B	B	B	D	D	D	i-b	ii-d	iii-a	iv-c

SOLUTIONS

Sol.1 - 2 (1-D, 2-D)

Since the same line is tangent at one point x = a and normal at other point x = b.

$\Rightarrow$ Tangent at x = b will be perpendicular to tangent at x = a.

$\Rightarrow$ Slope of tangent goes from positive to negative or negative to positive.

$\Rightarrow$ It takes the value zero some where.

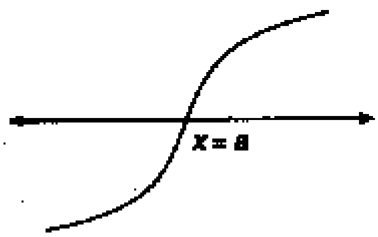
$\therefore$ There will be a point 'c' $\in$ (a, b), where $f'(c) = 0$.

Since f(x) is bijective, therefore x = c is neither a point of maximum nor of minimum.

Sol.3 (B)

According to graph,

$f(a + h) . f''(a - h) > 0$



Sol.4 (B)

According to above graph of f(a) = 0.

$\Rightarrow$ $f''(a) = 0$ (it is point of inflexion).

$\Rightarrow$ $f'''(a^r) = 0$, when $\alpha \le \gamma < \beta$.

(Using Rolle's theorem)

Sol.5 (B)

$f'(x) \neq 0$

$\Rightarrow$ f(x) = 0 has atmost one real root.

$\Rightarrow$ $f''(x) = 0$ has atmost one solution.

Sol.6 (D)

We have

$f(x) = x^2 + 2bx + 2c^2 = (x + b)^2 + 2c^2 - b^2$

$\Rightarrow$ min $f(x) = 2c^2 - b^2$

Also $g(x) = -x^2 - 2cx + b^2$

$= b^2 + c^2 - (x + c)^2$

so max $g(x) = b^2 + c^2$.

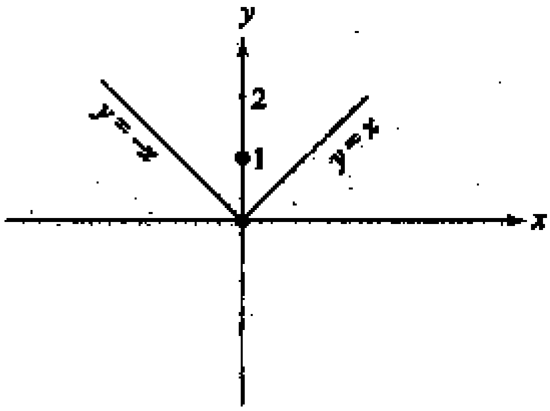
Since min f(x) > max g(x),

so $2c^2 - b^2 > b^2 + c^2$

$\Rightarrow$ $c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}\,|b|$

Sol.7 (D)

The graph of y = f(x) is given in fig. Here f(0) = 1, so the point (0, 0) is not on the graph of y = f(x). Thus at x = 0, the function f has neither local maximum nor local minimum.



Sol.8 (D)

We have $f(x) = 1 - \dfrac{2}{x^2 + 1}$

f(x) will be minimum if $2/(x^2 + 1)$ is maximum i.e. if $x^2 + 1$ is least i.e. when x = 0. Thus, minimum value of f(x) is f(0) = – 1.

Sol.9 (i-b, ii-d, iii-a, iv-c)

If f is as in (i), then f(x) = 1/2 (cos x – cos 3x). Since period of f is 2π and the function is even, it is sufficient to seek the greatest and least value among the interval $[0, \pi]$.

$f'(x) = (1/2)(3 \sin 3x - \sin x)$

The derivative vanishes at

$$x_1 = 0,\ x_2 = \cos^{-1}\frac{1}{\sqrt{3}},$$

$$x_3 = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right),\ x_4 = \pi$$

$$y(0) = y(\pi) = 0,\ y\left(\cos^{-1} \pm \frac{1}{\sqrt{3}}\right) = \pm\frac{4}{3\sqrt{3}}$$

Hence the greatest value is $\dfrac{4}{3\sqrt{3}}$ and the least value is $-\dfrac{4}{3\sqrt{3}}$. For function in (ii) greatest value of f(x) = f(2) = 8 and least value is f(2) = – 19. For function in (iii) greatest value is $f(0) = \dfrac{\pi}{2}$ and least value is $f\left(\pm\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{3}$. For (iv) greatest value is y(4) = 6 and the least value is y(0) = 0.