Dear student following is a Tough level [○ ○ ○ ● ○] test paper. Score of 12 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 Centre of a regular polygon of n sides is at origin and one vertex z_1 is known then vertex z_2 adjacent to z_1 is given by

(A) $z_1\left(\cos\frac{\pi}{n}+i\sin\frac{\pi}{n}\right)$ (B) $z_1\left(\cos\frac{\pi}{2n}\pm i\sin\frac{\pi}{2n}\right)$

(C) $z_1\left(\cos\frac{2\pi}{n}\pm i\sin\frac{2\pi}{n}\right)$ (D) None of these

Q.2 Roots of equation $z^n = (1 + z)^n$ lies on a
(A) Circle
(B) Line parallel to real axis
(C) Line parallel to imaginary axis
(D) None of these

Q.3 The value of the expression

$$\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n} \text{ equals } (n \geq 3)$$

(A) $\frac{n}{2}$ (B) $-\frac{n}{2}$ (C) $\frac{n}{3}$ (D) None

Q.4 The centre of square ABCD is at z = 0. If affix of the vertex A is z_1, then centroid of triangle ABC is-
(A) $z_1(\cos\pi \pm i\sin\pi)$

(B) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$

(C) $z_1\left(\cos\frac{\pi}{2}\pm i\sin\frac{\pi}{2}\right)$

(D) $\frac{z_1}{3}\left(\cos\frac{\pi}{2}\pm i\sin\frac{\pi}{2}\right)$

Q.5 If z is a complex number then minimum value of $|z| + |z - 2|$ is-

(A) 2 (B) $\frac{2+\sqrt{3}}{2}$ (C) $\frac{2-\sqrt{3}}{2}$ (D) None

Q.6 Let $z_1 = a + ib$, $z_2 = p + iq$ be two unimodular complex numbers such that $I_m.(z_1\bar{z}_2) = 1$. If $\omega_1 = a + ip$ and $\omega_2 = b + iq$ then-
(A) $R_e(\omega_1\omega_2) = 1$ (B) $I_m(\omega_1\omega_2) = 1$
(C) $R_e(\omega_1\omega_2) = 0$ (D) $I_m(\omega_1\bar{\omega}_2) = 1$

Q.7 The roots of the equation $1 + z + z^3 + z^4 = 0$ are represented by the vertices of -
(A) A square (B) An equilateral triangle
(C) A rhombus (D) None of these

Q.8 If ω is a complex cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively equal to-
(A) 0, 1 (B) 1, 1 (C) 1, 0 (D) −1, 1

Q.9 If a and b are two real numbers between 0 and 1 and points representing the complex numbers $z_1 = a + i$, $z_2 = 1 + ib$ alongwith origin form an equilateral triangle, then-

(A) $a = \sqrt{3}-1$, $b = \frac{\sqrt{3}}{2}$ (B) $a = 2-\sqrt{3} = b$

(C) $a = \frac{1}{2}$, $b = \frac{3}{4}$ (D) None of these

Q.10 Suppose z_1, z_2, z_3 are three vertices of an equilateral triangle circumscribing the circle $|z| = 1$. If $z_1 = 1 + \sqrt{3}i$ and z_1, z_2, z_3 are in anticlockwise sense then z_2 is-
(A) $1 - \sqrt{3}i$ (B) 2

(C) $\frac{1}{2}\left(1-\sqrt{3}i\right)$ (D) None of these

MATHEMATICS IIT JEE (JULY 1st WEEK CLASS TEST 5) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

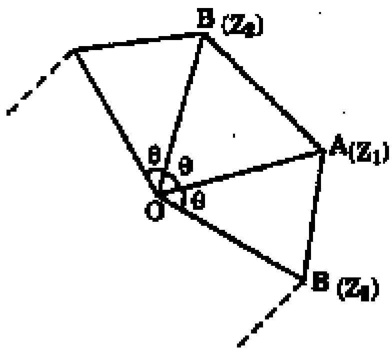
ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	B	D	A	D	B	B	B	D

SOLUTIONS

Sol.1 (C)

Each side of regular polygon of n sides subtends angle $\theta = \frac{2\pi}{n}$ at the centre.



Let the points A and B represent the vertices z_1 and z_2 and O is origin.

then OA = OB and $\angle BOA = \theta$. By rotation of OA about O through angle θ we have

$$\vec{OB} = \vec{OA}\ e^{\pm i\theta}$$

[± sign will be taken according θ is traced in anticlockwise or clockwise sense.]

$$\Rightarrow\ z_2 - 0 = (z_1 - 0)\,(\cos\theta \pm i\sin\theta)$$

or $$z_2 = z_1\left(\cos\frac{2\pi}{n} \pm i\sin\frac{2\pi}{n}\right)$$

Sol.2 (C)

$$z^n = (1 + z)^n$$

$$\Rightarrow \left(\frac{z+1}{z}\right)^n = 1$$

$$= \cos(2k\pi + 0) + i\sin(2k\pi + 0)$$

$$\Rightarrow 1 + \frac{1}{z} = \cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n}$$

where k = 0, 1, 2, ... , n − 1

$$\Rightarrow 1 - \cos\frac{2k\pi}{n} - i\sin\frac{2k\pi}{n} = -\frac{1}{z}$$

$$\Rightarrow 2\sin^2\frac{k\pi}{n} - 2i\sin\frac{k\pi}{n}\cos\frac{k\pi}{n} = -\frac{1}{z}$$

or $$-2\sin\frac{k\pi}{n}\left[i\cos\frac{k\pi}{n} - \sin\frac{k\pi}{n}\right] = -\frac{1}{z}$$

or $$z = \frac{1}{2\sin\frac{k\pi}{n}\left(i\cos\frac{k\pi}{n} - \sin\frac{k\pi}{n}\right)}$$

$$= \frac{-i}{2\sin\frac{k\pi}{n}\left(\cos\frac{k\pi}{n} + i\sin\frac{k\pi}{n}\right)}$$

or $$z = x + iy = \frac{-i}{2\sin\frac{k\pi}{n}}\left(\cos\frac{k\pi}{n} - \sin\frac{k\pi}{n}\right)$$

$$= \frac{1}{2}\left(-i\cot\frac{k\pi}{n} - 1\right) = -\frac{1}{2} - \frac{i}{2}\cot\frac{k\pi}{n}$$

$$\Rightarrow x = -\frac{1}{2} \text{ (constant)}$$

Thus roots lie on the line Re(z) = x = $-\frac{1}{2}$ which is parallel to imaginary axis.

Sol.3 (B)

Let $S \sum_{k=1}^{n-1} (n - k)\cos\frac{2\pi k}{n}$

$$= (n-1)\cos\frac{2\pi}{n} + (n-2)\cos\frac{4\pi}{n} + \ldots + 2\cos\frac{2(n-2)\pi}{n} + 1.\cos\frac{2(n-1)\pi}{n} \quad \ldots\ldots (1)$$

or $$S = 1.\cos\frac{2(n-1)\pi}{n} + 2\cos\frac{2(n-2)\pi}{n} + \ldots + (n-2)\cos\frac{4\pi}{n} + (n-1)\cos\frac{2\pi}{n}$$

(On reversing the order)

$$= 1.\cos\left(2\pi - \frac{2\pi}{n}\right) + 2.\cos\left(2\pi - \frac{4\pi}{n}\right) + \ldots + (n-2)\cos\frac{4\pi}{n} + (n-1)\cos\frac{2\pi}{n} + (n-1)\cos\left(2\pi - \frac{2(n-1)\pi}{n}\right)$$

$$= 1.\cos\frac{2\pi}{n} + 2.\cos\frac{4\pi}{n} + \ldots + (n-2)\cos\frac{2(n-2)\pi}{n} + (n-1)\cos\frac{2(n-1)\pi}{n} \quad \ldots\ldots (2)$$

Adding (1) and (2) we get

$$2S = n\left[\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+\cos\frac{6\pi}{n}+....+\cos\frac{2(n-1)\pi}{n}\right]$$

[(n − 1) terms of series with angles in A.P.]

$$= n.\frac{\cos\frac{1}{2}\left(\frac{2\pi}{n}+\frac{2(n-1)\pi}{n}\right).\sin\frac{(n-1)}{2}.\frac{2\pi}{n}}{\sin\frac{1}{2}\left(\frac{2\pi}{n}\right)}$$

$$= \frac{n.\cos\pi.\sin\left(\pi-\frac{\pi}{n}\right)}{\sin\frac{\pi}{n}}$$

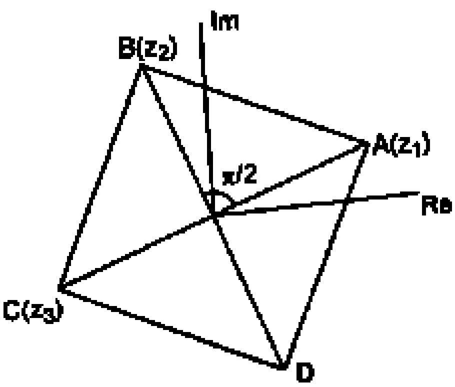
$$= n(-1) \qquad \Rightarrow \qquad S = -\frac{n}{2}$$

Sol.4 (D)

Point A represents z_1 and

OA = OB = OC = OD

and $\angle BOA = \angle COB = \frac{\pi}{2}$



Clearly C represents the complex number −z_1 and OB is obtained by rotating OA through angle $\frac{\pi}{2}$ either in clockwise or in anticlockwise sense.

Thus $\vec{OB} = \vec{OA}\, e^{\pm i\pi/2}$ or $z_2 = z_1 e^{\pm i\pi/2}$

or $z_2 = z_1\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$

and $C = z_3 = -z_1$

Centroid of ABC is

$$Z = \frac{z_1+z_2+z_3}{3}$$

$$= \frac{z_1 + z_1(\cos\pi.2 \pm i\sin\pi/2) - z_1}{3}$$

$$= \frac{z_1}{3}\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$$

Sol.5 (A)

We have $2 = |2| = |z - (z-2)|$

$\leq |z| + |z - 2|$

or $2 \leq |z| + |z-2|$

$\Rightarrow$ minimum value of $|z| + |z - 2|$ is 2.

Sol.6 (D)

$z_1\bar{z}_2 = (a + ib)(p - iq)$

$= (ap + bq) + i(bp - aq)$

Then $I_m(z_1\bar{z}_2) = 1 \Rightarrow bp - aq = 1$

Now $\omega_1\bar{\omega}_2 = (a + ip)(b - iq)$

$= (ab + pq) + i(bp - aq)$

Then $I_m(\omega_1\bar{\omega}_2) = bp - aq = 1.$

We can check that no other option hold good.

Sol.7 (B)

Given eq. is $(z + 1)(z^3 + 1) = 0$

or $(z + 1)(z + 1)(z^2 - z + 1) = 0$

$$\Rightarrow z = -1, -1\frac{1\pm i\sqrt{3}}{2}$$

Thus distinct roots are

$$z_1 = -1,\ z_2 = \frac{1}{2} + i\frac{\sqrt{3}}{2} = -\omega^2$$

and $z_3 = \frac{1}{2} - i\frac{\sqrt{3}}{2} = -\omega$

Let these be represented by points A, B and C.

Then $AB = |z_2 - z_1| = |1 - \omega^2|$

and $BC = |z_3 - z_2| = |\omega^2 - \omega|$

$= |\omega^2 - \omega^4|$

$= |\omega^2|\,|1 - \omega^2| = |1 - \omega^2|$

$= AB\ [\because |\omega^2| = 1]$

and $CA = |z_1 - z_3| = |\omega - 1|$

$= |\omega - \omega^3| = |\omega|\,|1 - \omega^2|$

$= AB$

Thus roots represent the vertices of an equilateral triangle.

Sol.8 (B)

Given $(1 + \omega)^7 = A + B$

$\Rightarrow (-\omega^2)^7 = A + B\omega$ as $1 + \omega = -\omega^2$

$\Rightarrow -\omega^{14} = -(\omega^3)^4 \cdot \omega^2 = A + B\omega$

$\Rightarrow -\omega^2 = A + B\omega$

$\Rightarrow -[-1 - \omega] = A + B\omega$

$\Rightarrow 1 + \omega = A + B\omega$

on comparing we get A = 1, B = 1

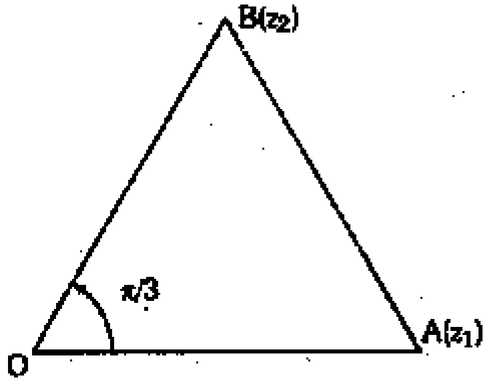
Sol.9 (B)

Let z_1 and z_2 be represented by points A and B. Then $\vec{OB}$ will be obtained by rotating $\vec{OA}$ by angle of $\frac{\pi}{3}$.

$\Rightarrow \vec{OB} = \vec{OA}\ e^{\pm i\pi/3}$ or $z_2 = z_1 e^{\pm i\pi/3}$

$\Rightarrow 1 + ib = (a + i)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)$

$\Rightarrow 1 + ib = \frac{1}{2}\left(a \mp \sqrt{3}\right) + \frac{i}{2}\left(1 \pm a\sqrt{3}\right)$



Taking first sign and comparing real and imaginary parts, we get

$1 = \frac{1}{2}\left(a - \sqrt{3}\right) \Rightarrow a = 2 + \sqrt{3} > 1$ thus we reject it. As given that $0 < a, b < 1$

Now taking second sign we get

$0 < a = 2 - \sqrt{3} < 1$

thus we take $a = 2 - \sqrt{3}$

From imaginary parts

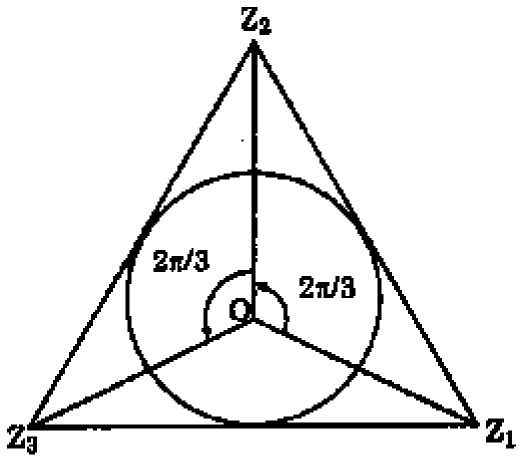
$$b = \frac{1}{2}\left(1 - a\sqrt{3}\right) = \frac{1}{2}\left[1 - \left(2 - \sqrt{3}\right)\sqrt{3}\right]$$

or $b = \frac{1}{2}\left(4 - 2\sqrt{3}\right) = 2 - \sqrt{3}$

Thus $a = 2 - \sqrt{3}$, $b = 2 - \sqrt{3}$.

Sol.10 (D)

$z_1 = 1 + \sqrt{3}\,i$ and centre of $|z| = 1$ is the origin, which is the incentre of the triangle. Point z_2 will be obtained by rotating oz_1, through angle of $\frac{2\pi}{3}$ in anticlockwise sense.



Thus $z_2 - 0 = (z_1 - 0)\ e^{2\pi i/3} = z_1\omega$

$\left[\because e^{2\pi i/3} = \omega\right]$

or $z_2 = \left(1 + \sqrt{3}i\right)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -2.$

Which is not given in any option.