Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one correct option)

Q.1 Let $[\sqrt{n^2+1}] = [\sqrt{n^2+\lambda}]$, where [.] is greatest integer function, $n, \lambda \in N$. Then λ can have
(A) 2n different values
(B) n different values
(C) 2n + 1 different values
(D) None of these

Q.2 Let $f(x) = \begin{cases} -1+\sin K_1\pi x, & x \text{ is rational} \\ 1+\cos K_2\pi x, & x \text{ is irrational} \end{cases}$

If f(x) is periodic function, then :
(A) either $K_1, K_2 \in$ rational or $K_1, K_2 \in$ irrational.
(B) $K_1, K_2 \in$ rational only.
(C) $K_1, K_2 \in$ irrational only.
(D) $K_1, K_2 \in$ irrational such that $\frac{K_1}{K_2}$ is rational.

Q.3 If $f : R \to \left[\frac{\pi}{6}, \frac{\pi}{2}\right)$, $f(x) = \sin^{-1}\left(\frac{x^2-a}{x^2+1}\right)$ is onto function, then set of values of 'a' is :
(A) $\left\{-\frac{1}{2}\right\}$ (B) $\left[-\frac{1}{2}, -1\right)$
(C) $(-1, \infty)$ (D) None of these

Q.4 Let $f(x) = \sin x + \cos(\sqrt{4-a^2})x$. Then the intergal values of 'a' for which f(x) is a periodic function are given by :
(A) {2, –2} (B) (–2, 2)
(C) [–2, 2] (D) None of these

Q.5 If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then g(f(x)) is invertible in the domain
(A) $\left[0, \frac{\pi}{2}\right]$ (B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (D) $[0, \pi]$

Q.6 Let $f : \mathbf{R} \to [-1, 1]$ is defined by $f(x) = \sin(2x + 1)$. If domain is restricted to

(i) $\left[-\frac{3\pi}{4}-\frac{1}{2}, -\frac{\pi}{2}-\frac{1}{2}\right]$	(a) f is one-one and onto
(ii) $\left[-\frac{3\pi}{4}-\frac{1}{2}, -\frac{1}{2}\right]$	(b) f is one-one but not onto
(iii) $\left[\frac{\pi}{4}-\frac{1}{2}, \frac{3\pi}{4}-\frac{1}{2}\right]$	(c) f is onto but not one-one
	(d) f is neither one-one onto

(A) (i)-(d), (ii)-(c), (iii)-(b)
(B) (i)-(b), (ii)-(c), (iii)-(a)
(C) (i)-(c), (ii)-(a), (iii)-(b)
(D) (i)-(b), (ii)-(c), (iii)-(d)

Please read short write up and answer subsequent questions (5 - 7)

For $x \neq 0, 1$ define
$f_1(x) = x$, $f_2(x) = 1/x$, $f_3(x) = 1 - x$,
$f_4(x) = 1/(1 - x)$, $f_5(x) = (x - 1)/x$, $f_6(x) = x(x - 1)$
This family of functions is closed under composition that is, the composition of any two of these functions is again one of these.

Q.7 Let F be a function such that $f_1 \circ F = f_4$. Then F is equal to
(A) f_1 (B) f_2 (C) f_3 (D) f_4

Q.8 Let G be a function such that $G \circ f_3 = f_6$. Then G is equal to
(A) f_5 (B) f_4 (C) f_3 (D) f_2

Q.9 Let H be a function such that $f_4 \circ H = f_5$. Then H is equal to
(A) f_2 (B) f_4 (C) f_5 (D) f_6

Q.10 Let J be a function such that $f_3 \circ J \circ f_2 = f_4$. Then J is equal to
(A) f_6 (B) f_5 (C) f_4 (D) f_3

MATHEMATICS IIT JEE (JUNE 4th WEEK CLASS TEST 1) (FUNCTIONS) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	C	D	B	B	D	A	B	C

SOLUTIONS

Sol.1 (A)

We have,

$n^2 + 1 = (n + 1)^2 - 2n < (n + 1)^2, n \in N$

i.e. $\sqrt{n^2+1} < n + 1$

or $n < \sqrt{n^2+1} < n + 1$

$\therefore$ $[\sqrt{n^2+\lambda}] = n$

$\Rightarrow$ $n < \sqrt{n^2+\lambda} < n + 1$

$\Rightarrow$ $n^2 < n^2 + \lambda < (n + 1)^2$

$\Rightarrow$ $0 < \lambda < 2n + 1$

$\Rightarrow$ λ can take 2n different value.

Sol.2 (B)

Range of $-1 + \sin K_1 \pi x$ is $[-2, 0]$ and range of $1 + \cos K_2 \pi x$ is $[0, 2]$

$\Rightarrow$ If $g(x) = -1 + \sin K_1 \pi x$

$\Rightarrow$ $g(x + T_1) = -1 + \sin K_1 \pi x$,

where T_1 is period of $1 + \sin K_1 \pi x$,

$\Rightarrow$ T_1 is rational

$\Rightarrow$ period of f(x) is rational.

$\Rightarrow$ K_1 and K_2 are rational.

Sol.3 (C)

Here, f(x) is onto

$\therefore$ $\frac{\pi}{6} \le \sin^{-1}\left(\frac{x^2-a}{x^2+1}\right) < \frac{\pi}{2}$

$\Rightarrow$ $\frac{1}{2} \le \frac{x^2-a}{x^2+1} < 1$

$\Rightarrow$ $\frac{1}{2} \le 1 - \frac{(a-1)}{x^2+1} < 1, \forall x \in R$

$\Rightarrow$ $a + 1 > 0$

$\Rightarrow$ $a \in (-1, \infty)$

Sol.4 (D)

f(x) will be periodic if $\sqrt{4-a^2}$ is a rational which is only possible when $(4 - a^2)$ is a perfect square.

$\Rightarrow$ $a = 0, 2, -2$ or $a \in \{-2, 0, 2\}$

Sol.5 (B)

Given $f(x) = \sin x + \cos x$ ....(1)

$g(x) = x^2 - 1$ ....(2)

Let $\phi(x) = g(f(x)) = g(\sin x + \cos x)$

$= (\sin x + \cos x)^2 - 1 = \sin 2x$

We know that

$\sin x$ is intertible if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\therefore$ $\sin 2x$ is invertible if $-\frac{\pi}{2} \le 2x \le \frac{\pi}{2}$

i.e. if $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$.

Sol.6 (B)

The function $f(x) = \sin(2x + 1)$ decreases from 1 to 0 on $\left[-\frac{3\pi}{4}-\frac{1}{2}, -\frac{\pi}{2}-\frac{1}{2}\right]$ so f is not onto but one-one side. Since $f\left(-\frac{\pi}{2}-\frac{1}{2}\right) = 0 = f\left(-\frac{1}{2}\right)$ so f is not one-one on $\left[-\frac{3\pi}{4}-\frac{1}{2}, -\frac{1}{2}\right]$ but is onto. On $\left[\frac{\pi}{4}-\frac{1}{2}, \frac{3\pi}{4}-\frac{1}{2}\right]$ f decreases from 1 to -1 and is continuous. Hence f is one-one and onto as well.

Solution 7 to 10

It is easy to see that

$f_1^{-1} = f_1, f_2^{-1} = f_2, f_3^{-1} = f_3, f_4^{-1} = f_5$.

Sol.7 (D)

$f_1 \text{ o } F = f_4$

$\Rightarrow$ $F = f_1^{-1} \text{ o } f_4 = f_1 \text{ o } f_4$.

Thus $F(x) = f_1(1/1 - x) = 1/(1 - x) = f_4(x)$

Sol.8 (A)

$G \text{ o } f_3 = f_6$

$\Rightarrow \quad G = f_6 \text{ o } f_3^{-1} = f_6 \text{ o } f_3$

So, $G(x) = f_6 (1 - x)$

$= (1 - x) / (1 - x) - 1$

$= x - 1 / x = f_5(x)$.

Sol.9 (B)

$f_4 \text{ o } H = f_5$

$\Rightarrow \quad H = f_4^{-1} \text{ o } f_5 = f_5 \text{ o } f_5$. Therefore,

$H(x) = f_5 (f_5(x))$

$= f_5 ((x - 1)/x)$

$$= \frac{\frac{x-1}{x} - 1}{(x-1)/x}$$

$$= \frac{1}{1-x} = f_4(x)$$

Sol.10 (C)

$J = f_3^{-1} \text{ o } f_4 \text{ o } f_2^{-1} = f_3 \text{ o } f_4 \text{ o } f_2$.

Thus $J(x) = f_3 \text{ o } f_4 (1/x) = f_3 (1/1 - 1/x)$

$= f_3 (x/x - 1)$

$= 1 - x/x - 1 = 1/1 - x = f_4(x)$.