Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one option correct)

Q.1 Domain of definition of the function

$$y = \sqrt{\cos(\sin x)} + \sin^{-1}\frac{1+x^2}{2x}$$

(A) {–1, 1} (B) {0}
(C) R (D) R – {–1, 1}

Q.2 If f(x) is continuous function and domain of $g(x) = \sqrt{f(x) - x}$ be R and $h(x) = \frac{5x+4}{2x-5}$, then domain of $\phi(x) = \sqrt{f(f(x)) - h(h(x))}$ is

(A) R–{2/5} (B) R – {5/2}
(C) R (D) None

Q.3 If $f(x) = x - [x]$, $K \le x < K + 0.5$

$= [x]$, $K + 0.5 \le x < K + 1$, $K \in I$

and $g(x) = \sin^4 x + \cos^4 x$, then f(g(x)) =

(A) 0 (B) 1 (C) 2 (D) –1

Q.4 For what values of x the functions

$f(x) = \frac{x^2}{x}$ and g (x) = x are identical

(A) R (B) R – {0}
(C) $[0, \infty)$ (D) $(-\infty, 0]$

Q.5 Let f(x,y) be a periodc function satisfying $f(x,y) = f(2x + 2y, 2y - 2x)\ \forall x,y$.
let $g(x) = f(2^x, 0)$, then g(x) is periodic function with period

(A) 2 (B) 12
(C) 8 (D) Non periodic

Q.6 The domain of dervative of the function $f(x) = |\sin^{-1}(2x^2 - 1)|$, is :

(A) (–1, 1) (B) $(-1, 1) \sim \left\{0, \pm\frac{1}{\sqrt{2}}\right\}$

(C) (–1, 1) ~ {0} (D) $(-1, 1) \sim \left\{\pm\frac{1}{\sqrt{2}}\right\}$

Q.7 If domain for y = f (x) is [–3, 2], then domain of g (x) = f {|[x]|} =

(A) [–2, 3) (B) [–2, 3]
(C) (–2, 3) (D) (–2, 3]

Q.8 Let [x] be the greatest integer less than or equal to x. Then the equation sinx = [1 + sinx] + [1 – cosx] has :

(A) one solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) one solution in $\left[\frac{\pi}{2}, \pi\right]$

(C) one solution in R (D) no solution in R

Q.9 The number of solutions of |[x] –2x| = 4, where [x] is the greatest integer ≤ x.

(A) 1 (B) 2 (C) 3 (D) 4

Q.10 The range of

$$f(x) = \frac{1}{\pi}(\sin^{-1}x + \tan^{-1}x) + \frac{x+1}{x^2+2x+5} \text{ is :}$$

(A) $\left[-\frac{3}{4}, \frac{1}{5}\right]$ (B) $\left[-\frac{5}{4}, \frac{3}{4}\right]$

(C) $\left[-\frac{3}{4}, \frac{5}{4}\right]$ (D) $\left[-\frac{3}{4}, 1\right]$

MATHEMATICS IIT JEE (JUNE 4th WEEK CLASS TEST 2) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	A	B	B	B	A	D	D	D

SOLUTIONS

Sol.1 (A)

For y to be defined

(i) $\cos(\sin x) \geq 0 \Rightarrow -\infty < x < \infty$ ... (1)

Let $\theta = \sin x$, then $-1 \leq 0 \leq 1$

$\therefore$ $-58^0 < \theta < 58^0$

[$\because$ 1 radian = 57.75^0 (approx.)]

$\Rightarrow$ $\cos\theta > 0$

Thus cos (sinx) > 0 for all real x

(ii) $\sin^{-1}\frac{1+x^2}{2x}$ must be defined

$\Rightarrow \left|\frac{1+x^2}{2x}\right| \leq 1 \quad \Rightarrow \quad \frac{|1+x^2|}{2|x|} \leq 1$

$\Rightarrow$ $1 + x^2 \leq 2|x|$ [$\because$ $|x| > 0$]

$\Rightarrow$ $1 + |x|^2 - 2|x| \leq 0$

$\Rightarrow$ $(1 - |x|)^2 \leq 0$ $\Rightarrow$ $(1 - |x|)^2 = 0$

$\Rightarrow$ $|x| = 1$ $\Rightarrow$ $x = \pm 1$ ...(2)

From (1) and (2), common values of x are given by x = −1, 1

$\because$ Domain of y = {−1, 1}.

Sol.2 (B)

$h(x) = \frac{5x+4}{2x-5}$

h(h(x)) = h(y), where y = h(x)

$$= \frac{5y+4}{2y-5} = \frac{5\left(\frac{5x+4}{2x-5}\right)+4}{2\left(\frac{5x+4}{2x-5}\right)+4}$$

$$= \frac{33x}{33} = x,\ x \neq \frac{5}{2}$$

$\because$ domain of g(x) is R

$\therefore$ $f(x) - x \geq 0\ \forall\ x \in R$

$\therefore$ $f(f(x)) - f(x) \geq 0$

$\Rightarrow$ $f(f(x)) - x \geq 0$

$\Rightarrow$ $f(f(x)) - h(h(x)) \geq 0\ \forall\ x \in R,\ x \neq \frac{5}{2}$

$\therefore$ domain of $\phi(x) = R - \frac{5}{2}$

Sol.3 (A)

given $f(x) = x - K,\quad K \leq x < K + 0.5$

$= K,\quad K + 0.5 \leq x < x + 1$

and $g(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$= 1 - \frac{1}{2}\sin^2 2x$

clearly $\frac{1}{2} \leq g(x) \leq 1$

g(x) = 1 when sin 2x = 0 i.e. $x = \frac{n\pi}{2}$, $n \in I$

$\Rightarrow$ When $\frac{1}{2} \leq g(x) < 1$, f(x) = 0 [here x= 0]

When g(x) = 1, f(g(x) = f(1) = 1 − [1] =0

Thus $f(g(x)) = 0\ \forall\ x \in R$.

Sol.4 (B)

Domain of f = R − {0} = $(-\infty, 0) \cup (0, \infty)$

and Domain of g = R

$\Rightarrow$ f (x)and g (x) are identical $\forall\ x \neq 0$

Sol.5 (B)

Given f (x,y) = f (2x + 2y, 2y − 2x)

$\therefore$ f (x,y) = f(2x + 2y, 2y − 2x)

= f [2 (2x + 2y + 2 (2y − 2x),

2(2y − 2x) −2(2x + 2y)]

= f (8y, −8x)

f(x,y) = f (8y, −8x)

= f (−64x, −64y)

= f [(−64) (−64x), (−64) (−64y)]

$= f[2^{12}x, 2^{12}y]$

$\therefore$ $f(x, 0) = f(2^{12}x, 0)$

Now $g(x) = f(2^x, 0)$

$= f(2^{12} . 2^x, 0)$

$= f(2^{x+12}, 0)$

= g (x + 12)

$\Rightarrow$ g(x) is a periodic function with period 12.

Sol.6 (B)

$y = |\sin^{-1}(2x^2 - 1)|$,

$\therefore \quad \frac{dy}{dx} \frac{|\sin^{-1}(2x^2-1)|}{\sin^{-1}(2x^2-1)} \cdot \frac{4x}{|2x^2|\sqrt{1-x^2}}$

which would exists if,

$|2x| \neq 0, \sin^{-1}(2x^2-1) \neq 0$ and $1 - x^2 > 0$

$\Rightarrow \quad x \neq 0, 2x^2 - 1 \neq 0$ and $|x| < 1$

$\Rightarrow \quad x \neq 0, \pm\frac{1}{\sqrt{2}}$ and $|x| < 1$

$\Rightarrow \quad x \in (-1, 1) \sim \left\{0, \pm\frac{1}{\sqrt{2}}\right\}$

Sol.7 (A)

Here, f(x) is defined by [−3, 2]

$\Rightarrow \quad x \in [-3, 2]$.

(i.e. the only value of x we can substitute lie between [−3, 2]).

For g (x) = f {|[x]|} to be defined, we must have

$-3 \leq |[x]| \leq 2$

$\Rightarrow \quad 0 \leq |[x]| \leq 2 \quad [\text{as} |x| \geq 0 \text{ for all } x]$

$\Rightarrow \quad -2 \leq [x] \leq 2 \quad [\text{as} |x| \leq a \Rightarrow -a \leq x \leq a]$

$\Rightarrow \quad -2 \leq x < 3$

[by definition of greatest integral function].

Hence, domain g (x) $\in$ [−2, 3[or [−2, 3).

Sol.8 (D)

Given that [1 + sinx] + [1−cosx] = sinx

$\Rightarrow$ 1+ [sinx] + 1 + [−cosx] = sinx

$\Rightarrow$ 2+ [sinx] + [−cosx] = sinx

$\Rightarrow$ 2 + [−cosx] = {sinx}

Here, LHS is 1,2, or 3 but RHS $\in$ [0, 1)

$\therefore$ no solution.

Sol.9 (D)

Let x = I + f, I $\in$ integer, f $\in$ fractional part (i.e. $0 \leq f < 1$)

$|[x] - 2x| = 4$

$\Rightarrow \quad |[I + f] - 2(I + f)| = 4$

$\Rightarrow \quad |I - 2I - 2f| = 4$

$\Rightarrow \quad |I + 2f| = 4$,

which is only possible if, $\left(f = \frac{1}{2} \text{ or } 0\right)$

If $\quad f = \frac{1}{2}$

$\Rightarrow \quad |I + 1| = 4$

$\Rightarrow \quad I + 1 = \pm 4$

So, $\quad I = 3, -5$ and $f = \frac{1}{2}$

If $\quad f = 0$,

Then $\quad |I| = 4$

$I = \pm 4$ and $f = 0$

Thus number of solutions are

$x = \left\{\pm 4, \frac{7}{2}, -\frac{9}{2}\right\}$ i.e. 4 solutions.

Sol.10 (D)

Here,

$f(x) = \frac{1}{\pi}(\sin^{-1}x + \tan^{-1}x) + \frac{1}{(x+1) + \frac{4}{(x+1)}}$

$= g(x) + h(x)$

where, domain of g(x) $\in$ [−1, 1]

$\therefore \quad$ Maximum value of $g(x) = g(1) = \frac{3}{4}$

and minimum value of $g(x) = g(-1) = -\frac{3}{4}$

Also, maximum value of h(x) occurs when $(x + 1) + \frac{4}{(x+1)}$ is minimum at x = 1.

$\Rightarrow \quad$ Range of $f(x) \in \left[-\frac{3}{4}, 1\right]$