Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one option correct)

Q.1 If [x] denotes the integral part of x and f(x) $= \dfrac{\sin^3 x}{\left[\dfrac{x}{\pi}\right]+\dfrac{1}{2}}$, then f(x) is-

(A) An odd function
(B) An even function
(C) Neither even nor odd
(D) None of these

Q.2 If A and B are the points of intersection of $y = f(x)$ and $y = f^{-1}(x)$, then-

(A) A and B necessarily lie on the line y = x
(B) A and B must be coincident
(C) Slope of line AB may be – 1
(D) None of these

Q.3 $f(x) = x^3 + 3x^2 + 4x + b \sin x + c \cos x$, $\forall\ x \in R$ is a one-one function then the value of $b^2 + c^2$ is :

(A) ≥ 1 (B) ≥ 2
(C) ≤ 1 (D) None of these

Q.4 If $f(x) = \tan^2 \dfrac{\pi x}{n^2 - 5n + 8} + \cot (n + m)\pi x$; ($n \in N$, $m \in Q$), is a periodic function with 2 as its fundamental period, then m can't belong to :

(A) $(-\infty, -2) \cup (-1, \infty)$
(B) $(-\infty, -3) \cup (-2, \infty)$
(C) $(-2, -1) \cup (-3, -2)$
(D) $\left(-3, -\dfrac{5}{2}\right) \cup \left(-\dfrac{5}{2}, -2\right)$

Q.5 Let f(x) be a periodic function with period 3 and $f\left(-\dfrac{2}{3}\right) = 7$ and $g(x) = \int_0^x f(t + n)\, dt$, where n = 3K, $K \in N$. Then $g'\left(\dfrac{7}{3}\right)$ is equal to-

(A) $-\dfrac{2}{3}$ (B) 7 (C) –7 (D) $\dfrac{7}{3}$

(Q.No. 6 to 10)
Given an odd function f defined and integrable everywhere and periodic with period 2. Let $g(x) = \int_0^x f(t)\, dt$
Answer the following questions :

Q.6 f(4) is equal to-

(A) 0 (B) 2
(C) 4 (D) None of these

Q.7 g(4) is equal to-

(A) 0 (B) 2
(C) 4 (D) None of these

Q.8 g(x + 2) is equal to-

(A) g(x) g(2) (B) g(x) + g(2)
(C) g(x) – g(2) (D) None of these

Q.9 If f'(–2) = – 2, then what should be f'(2) ?

(A) 0 (B) –2
(C) 2 (D) None of these

Q.10 If $g(x^2) = x^2(1 + x)$, then roots of the equation $x^2 - f(x^2) = 0$ are-

(A) $-\dfrac{1}{2}, 2$ (B) $\dfrac{1}{2}, 2$
(C) $\dfrac{1}{2}, -2$ (D) $-\dfrac{1}{2}, -2$

MATHEMATICS IIT JEE (JUNE 4th WEEK CLASS TEST 3) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	C	C	B	A	A	B	B	A

SOLUTIONS

Sol.1 (B)

Given, $f(x) = \dfrac{\sin^3 x}{\left[\dfrac{x}{\pi}\right] + \dfrac{1}{2}}$ ...... (1)

$\therefore\quad f(-x) = -\dfrac{\sin^3 x}{\left[-\dfrac{x}{\pi}\right] + \dfrac{1}{2}}$ ...... (2)

Case I : When $x \neq n\pi,\ n \in I$

In this case $\dfrac{x}{\pi} \neq$ an integer

$\therefore\quad \left[\dfrac{x}{\pi}\right] + \left[-\dfrac{x}{\pi}\right] = -1$

$\Rightarrow\quad \left[\dfrac{x}{\pi}\right] = -1 - \left[\dfrac{x}{\pi}\right]$

$\Rightarrow\quad \left[\dfrac{x}{\pi}\right] + \dfrac{1}{2} = -\left(\left[\dfrac{x}{\pi}\right] + \dfrac{1}{2}\right)$

$\therefore$ from (2), $f(-x) = \dfrac{\sin^3 x}{\left[\dfrac{x}{\pi}\right] + \dfrac{1}{2}} = f(x)$ ....(A)

Case II : When $x = n\pi,\ n \in I$

In this case $\dfrac{x}{\pi} = n,\ n \in I$

$\therefore\quad \left[\dfrac{x}{\pi}\right] = n$

$\therefore\quad f(x) = \dfrac{\sin^3 n\pi}{n + \dfrac{1}{2}} = 0$

and $f(-x) = \dfrac{\sin^3 n\pi}{-n + \dfrac{1}{2}} = 0$

$\therefore\quad f(-x) = f(x)$ .... (B)

From (A) and (B), in all cases $f(-x) = f(x)$, hence $f(x)$ is an even function.

Sol.2 (C)

If solution of $f(x) = f^{-1}(x)$ doesn't lie on $y = x$ then they must be of the form (α, β) and (β, α).

$\therefore$ Slope of line AB may be -1.

Sol.3 (C)

Here, $f(x) = x^3 + 3x^2 + 4x + b \sin x + c \cos x$

$\Rightarrow f'(x) = 3x^2 + 6x + 4 + b \cos x - c \sin x$

Now for $f(x)$ to be one-one only possibility is

$f'(x) \geq 0,\ \forall\ x \in R$

i.e., $3x^2 + 6x + 4 + b \cos x - c \sin x \geq 0,\ \forall\ x \in R$

i.e., $3x^2 + 6x + 4 \geq c \sin x - b \cos x,\ \forall\ x \in R$

i.e., $3x^2 + 6x + 4 \geq \sqrt{b^2 + c^2},\ \forall\ x \in R$

i.e., $\sqrt{b^2 + c^2} \leq 3(x^2 + 2x + 1) + 1,\ \forall\ x \in R$

$\sqrt{b^2 + c^2} \leq 3(x + 1)^2 + 1,\ \forall\ x \in R$

$\sqrt{b^2 + c^2} \leq 1,\ \forall\ x \in R$

$\Rightarrow\quad b^2 + c^2 \leq 1,\ \forall\ x \in R$

Sol.4 (C)

Period is LCM of $n^2 - 5n + 8$ and $\dfrac{1}{n+m}$

$\Rightarrow\quad n^2 - 5n + 8 = 1$ or $n^2 - 5n + 8 = 2$

$\Rightarrow\quad n^2 - 5n + 7 = 0$ or $n^2 - 5n + 6 = 0$

Since, $n \in N$, $\therefore$ $n = 2, 3$

and $\left(\dfrac{1}{n+m}\right) K_1 = 2,\ (K_1 \in I)$

$\Rightarrow\quad \dfrac{1}{n+m} \not> 1$

$\therefore\quad \dfrac{1}{2+m} \not> 1$ or $\dfrac{1}{3+m} \not> 1$

$\Rightarrow\quad m \notin (-2, -1) \cup (-3, -2)$

Sol.5 (B)

$g'(x) = f(x + n) = f(x + (n - 3) + 3)$

$= f(x + n - 3)$

$= ....$

$................$

$= f(x + 3) = f(x)$

$\Rightarrow g'(x) = f(x)$

$\Rightarrow g'\left(\frac{7}{3}\right) = f\left(\frac{7}{3}\right) = f\left(-\frac{2}{3}+3\right) = f\left(-\frac{2}{3}\right) = 7$

Sol.6 (A)

Since, f is an odd function and defined everywhere $\Rightarrow f(0) = 0$

$f(4) = f(2) = f(0) = 0$ ($\because$ f is periodic with period 2)

Sol.7 (A)

$g(4) = \int_0^4 f(t)\,dt$

Put $t - 2 = u$

$\Rightarrow g(4) = \int_{-2}^{2} f(u + 2)\,du$

$= \int_{-2}^{2} f(u)\,du$ [$\because$ f is period with period 2, $f(u + 2) = f(u)$]

$= 0$ [$\because$ f is an odd function]

Sol.8 (B)

$g(x + 2) = \int_0^{x+2} f(t)\,dt$

$= \int_0^{x} f(t)\,dt + \int_x^{x+2} f(t)\,dt$

$= g(x) + \int_0^{2} f(t)\,dt = g(x) + g(2)$

Sol.9 (B)

Since, f is an odd function, graph of f(x) in (0, 2) and (–2, 0) will be symmetry of each other about origin.

$\Rightarrow$ Slope of tangent for function f at $x = -2$ and $x = 2$ will be same

$\Rightarrow f'(-2) = f'(2) = -2$

Sol.10 (A)

Given, $g(x^2) = x^2(1 + x)$

$\Rightarrow 2xg'(x^2) = 2x + 3x^2$;

$\Rightarrow g'(x^2) = 1 + \frac{3}{2}x$

Now $g'(x) = f(x)$ (by the given relation)

$\Rightarrow f(x^2) = 1 + \frac{3}{2}x$

So equation $x^2 - f(x^2) = 0$ becomes,

$x^2 - 1 - \frac{3}{2}x = 0$

$\Rightarrow 2x^2 - 3x - 2 = 0 \Rightarrow x = -\frac{1}{2}, 2.$