*Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one option correct)*

**Q.1** Let n be a +ve integer and define f(n) = 1! + 2! + 3! + ....... + n!, then values of P(x) and Q(x) such that f(n + 2) = Q(n) f(n) + P(n) f(n + 1) for all n $\geq$ 1 are-
(A) P(x) = x – 3 (B) P(x) = – x + 3
(C) Q(x) = – x – 2 (D) Q(x) = x + 2

**Q.2** If $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$ a > 0, then $\sum_{r=1}^{2n-1} 2f\left(\dfrac{r}{2n}\right) =$
(A) 2n (B) 2n + 1 (C) 2n – 1 (D) n

**Q.3** Let S(n) denote the number of ordered pairs (x, y) satisfying $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{n}$ where n > 1 and x, y, n $\in$ N, then the value of S(6) =
(A) 3 (B) 6 (C) 9 (D) 12

**Q.4** If a + b + c = abc, a, b and c $\in$ $R^+$, then a + b + c
(A) $\geq \sqrt{3}$ (B) $\dfrac{1}{\sqrt{3}}$ (C) $3\sqrt{3}$ (D) 3

**Q.5** The integral solution for $n_1n_2 = 2n_1 - n_2$, where $n_1n_2 \in$ I is/are-
(A) 3 (B) 0 (C) 4 (D) 2

**Q.6** If 2 f(xy) = $\{f(x)\}^y + \{f(y)\}^x$ for all x, y $\in$ R, and f(1) = 2 then find the value of $\sum_{r=1}^{100} f(r)$
(A) $2^{101}$ (B) $2^{101} - 2$
(C) $2^{100}$ (D) $2^{100} - 1$

**(Q.No.7 to 10) :** The accompanying figure shows the graph of a function f(x) with domain [0, 2] and range [0, 1]

Fig. (a) :

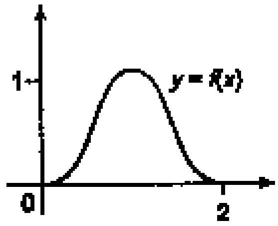



Fig. (b) :

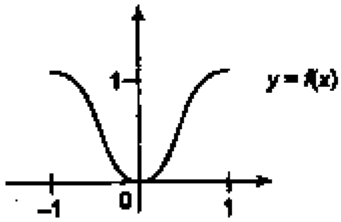
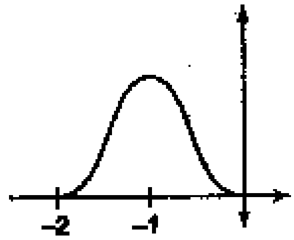



Fig. (c) :




**Q.7** Fig. (a) represents the graph of the function
(A) – f(x) (B) –f(x – 1) + 1
(C) – f(x + 1) – 1 (D) – f(x + 1) + 1

**Q.8** [1, 3] and [0, 1] are the domain and range (respectively) of the function
(A) – f(x) (B) f(x – 1)
(C) – f(x + 1) + 1 (D) –f(x + 1)

**Q.9** Fig. (c) represents the graph of function -
(A) 2f(x) (B) f(x – 2)
(C) f(x + 2) (D) f(x – 2) + 1

**Q.10** The domain and range (respectively) of
(A) f(–x) are [–2, 0] and [–1, 0]
(B) f(x) – 1 are [0, 2] and [0, 1]
(C) f(x) + 2 are [0, 2] and [1, 2]
(D) –f(x + 1) + 1 are [–1, 1] and [0, 1]

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## MATHEMATICS IIT JEE (JUNE 4th WEEK CLASS TEST 4) (FUNCTIONS) ANSWER KEY

**Name :** ..................................................................................... **Roll No. :** .................................

| | A | B | C | D | | A | B | C | D | | A | B | C | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ○ | ○ | ○ | ○ | 4 | ○ | ○ | ○ | ○ | 7 | ○ | ○ | ○ | ○ |
| 2 | ○ | ○ | ○ | ○ | 5 | ○ | ○ | ○ | ○ | 8 | ○ | ○ | ○ | ○ |
| 3 | ○ | ○ | ○ | ○ | 6 | ○ | ○ | ○ | ○ | 9 | ○ | ○ | ○ | ○ |
| | | | | | | | | | | 10 | ○ | ○ | ○ | ○ |

## ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | C | C | C | C | C | B | D | D | C | D |

# SOLUTIONS

**Sol.1 (C)**

$f(n) = 1! + 2! + 3! + ..... + n!$

$f(n + 2) = 1! + 2! + 3! + .... + (n + 2)!$

$\Rightarrow \{1! + 2! + ...... + (n + 2)!\}$
$= Q(n) \{1! + 2! + 3! + .... + n!\}$
$+ P(n) \{1! + 2! + ...... + (n + 1)!\}$

$\Rightarrow \{1! + 2! +....+ n!\} + (n + 1)! + (n + 2)!$
$= (Q(n) + P(n) \{1! + 2! + ....... + n!\}$
$+ P(n)(n + 1)!$

Equating coefficients of $\{1! + 2! + .... + n!\}$ and $(n + 1)!$ on both sides, we get

$Q(n) + P(n) = 1$ and $P(n) = (n + 3)$

On simplify, we get

$Q(n) = - n - 2$ or $Q(x) = - x - 2$

and $P(x) = x + 3$

**Sol.2 (C)**

Given $f(x) = \frac{a^x}{a^x + \sqrt{a}}$ ..... (1)

$\therefore \quad f(1 - x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}} = \frac{\sqrt{a}}{\sqrt{a} + a^x}$ ...(2)

$\therefore$ from (1) & (2) we have

$f(x) + f(1 - x) = 1$

$$\Rightarrow \sum_{r=1}^{2n-1} f\left(\frac{r}{2n}\right) + \sum_{r=1}^{2n-1} f\left(\frac{2n-r}{2n}\right) = 2n - 1$$

Let $2n - r = t$

$$\Rightarrow \sum_{r=1}^{2n-1} f\left(\frac{r}{2n}\right) + \sum_{t=1}^{2n-1} f\left(\frac{t}{2n}\right) = 2n - 1$$

$$\Rightarrow 2\sum_{r=1}^{2n-1} f\left(\frac{r}{2n}\right) = 2n - 1$$

**Sol.3 (C)**

Given $nx + ny = xy$

or $xy - nx - ny + n^2 = n^2$

or $(x - n)(y - n) = n^2$

$\Rightarrow$ $(x - n)(y - n)$ are two integral factors of $n^2$

Obviously if d is one divisor of $n^2$ then for each such divisor there will be an ordered pair (x, y)

$\Rightarrow$ S(n) = number of divisors of $n^2$

$\therefore$ for n = 6 we have d = 1, 2, 3, 6, 9, 12, 18, 36

Thus S(6) = 9

**Sol.4 (C)**

Using A.M. $\geq$ G.M.

We have $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

$\frac{a+b+c}{3} \geq (a + b + c)^{1/3}$

given $a + b + c = abc$

$\Rightarrow (a + b + c)^{2/3} \geq 3$

$\Rightarrow (a + b + c) \geq 3\sqrt{3}$

**Sol.5 (C)**

$n_1n_2 = 2n_1 - n_2$

$\Rightarrow n_2(n_1 + 1) = 2n_1 \Rightarrow n_2 = \frac{2n_1}{n_1+1}$

$\Rightarrow n_2 = 2 - \frac{2}{n_1+1}$

$\therefore \frac{2}{n_1+1} \in$ integer

or $n_1 + 1 = - 2, -1, 1, 2$

or $n_1 = - 3, -2, 0, 1$

$\Rightarrow n_2 = 3, 4, 0, 1$

**Sol.6 (B)**

Given, $2f(xy) = \{f(x)\}^y + \{f(y)\}^x$

for all $x, y \in R$ ........ (1)

Putting y = 1, we get $2f(x) = f(x) + \{f(1)\}^x$

$\therefore \quad f(x) = 2^x \qquad \therefore \quad f(r) = 2^r$

Now $\sum_{r=1}^{100} f(r) = \sum_{r=1}^{100} 2^r = 2 + 2^2 + 2^3 + .... + 2^{100}$

$= \frac{2(2^{100}-1)}{2-1} = 2^{101} - 2.$

**Sol.7-10 [7-D, 8-D, 9-C, 10-D ]**

To shift the graph of a function $y = f(x)$ straight up, we add a positive constant. To shift the graph of a function $y = f(x)$ straight down, we add a negative constant to the right hand side of $y = f(x)$. To shift the graph of $y = f(x)$ to the left, we add a positive constant to x and to shift to the right, we add a negative constant to x.

For Q. 7, sin the fig. (a) is inversion then shifting down and then shifting to the left so it $-f(x + 1) + 1$.

Q. 8, 9 are clear.

Q.10 We find that the domain and range of $-f(x + 1) + 1$ are $[-1, 1]$ and $[0, 1]$.