Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-7 (+3,–1). (Questions may have more than one options correct)

Q.1 The period of the function

$$f(x) = \text{cosec}^2 3x + \cot 4x \text{ is}$$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) π

Q.2 Which of the following sets of ordered pairs define a one to one function ?

(A) $R = \{(x, y); x^2 + y^2 = 2\}$ on **R**

(B) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$ and $R = \{(x, y) : 5x + 2y$ is a prime number$\}$ on A

(C) $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(x, y) : y = x^2 - 3x + 3\}$ on A

(D) None of these

Q.3 Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = g(x) + h(x)$, where $g(x)$ is an even function and $h(x)$ is an odd function. Then the value of the $g(x + y) + g(x - y)$ is

(A) $2g(x)\, g(y)$ (B) $2g(x + y)\, g(x - y)$

(C) $2g(x)$ (D) None of these

Q.4 The domain of definition of

$$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2 - 36} \text{ is}$$

(A) $(-\infty, 0) \sim \{-6\}$ (B) $(0, \infty) \sim \{1, 6\}$

(C) $(1, \infty) \sim \{6\}$ (D) $[1, \infty) \sim \{6\}$

Q.5 Let f be a function defined on [0, 2], then the function $g(x) = f(9x^2 - 1)$ has domain

(A) [0, 2] (B) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

(C) [–3, 3] (D) None of these

Q.6 Which of the following defines a one - one function ?

(A) $f(x) = e^x$ $(-\infty < x < \infty)$

(B) $f(x) = e^{x^2}$ $(-\infty < x < \infty)$

(C) $f(x) = \cos x$ $(0 \le x \le \pi)$

(D) $f(x) = ax + b$ $(-\infty < x < \infty)$ $(a \ne 0)$

Q.7 Which of the following functions are periodic

(A) $f(x) = \{x\}$, the fractional part of the number x

(B) $f(x) = |\cos x|$

(C) $f(x) = 1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$

(D) $f(x) = x + \sin x$

MATHEMATICS IIT JEE (JUNE 3rd WEEK CLASS TEST 1) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○					
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	D	D	A	C	D	A, C, D	A, B, C

SOLUTIONS

Sol.1 (D)

$f(x) = 2/(1 - \cos 6x) + 1/(\tan 4x)$.

So the period of f = l. c.m. (period of cos 6x, period of tan 4x) = l.c.m. $\left(\frac{\pi}{3}, \frac{\pi}{4}\right) = \pi$.

Sol.2 (D)

Since (1, –1) and (1, 1) $\in$ R in (a), so R cannot define a function. Also (1, 1) and (1, 3) $\in$ R in (b). So R doesn't define a function in this case also. Moreover, (1, 2), (2, 1) $\in$ R in (c). So R in (c) also cannot define a one-to-one function.

Sol.3 (A)

Clearly $g(x) = \frac{1}{2}(a^x + a^{-x})$ and

$$h(x) = \frac{1}{2}(a^x + a^{-x})$$

Now

$g(x + y) + g(x - y)$

$= \frac{1}{2}(a^{x+y} + a^{-(x+y)}) + \frac{1}{2}(a^{x-y} + a^{-x+y})$

$= 2.\frac{1}{4}(a^x a^y + a^x a^{-y} + a^{-x} a^y + a^{-x} a^{-y}$

$= 2.\frac{1}{4}(a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})$

$= 2.\left(\frac{1}{2}(a^x + a^{-x})\right)\left(\frac{1}{2}(a^y + a^{-y})\right)$

$= 2.g(x)\, g(y)$

Sol.4 (C)

For f to be defined $x \neq -6$, 6 and $\log_{0.4}\left(\frac{x-1}{x+5}\right) \geq 0$, $\frac{x-1}{x+5} > 0$. Since $\log_a x$ for $0 < a < 1$ is a decreasing function, we have $\frac{x-1}{x+5} \leq 1$ and $\frac{x-1}{x+5} > 0$. For $x > -5$, we must have $x - 1 \leq x + 5$ and $x - 1 > 0$. The first inequality is always ture, so we must have $x > 1$. For $x < -5$, we have $x - 1 \geq x + 5$, $x - 1 < 0$. These inequalities are not possible. Hence the domain of f is $(1, \infty) \sim \{6\}$.

Sol.5 (D)

g is meaningful is

$$0 \leq 9x^2 \leq 2 \Leftrightarrow 1 \leq 9x^2 \leq 2$$

i.e. $x \in \left[\left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)\right] \cap \left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

$= \left[-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$.

Sol.6 (A, C, D)

If $f(x) = e^x$ then $f(x_1) = f(x_2)$

$\Rightarrow \quad e^{x1} = e^{x2}$

$\Rightarrow \quad e^{x1} = e^{x2} = 1$

$\Rightarrow \quad x_1 = x_2 = 0$

$\Rightarrow \quad x_1 = x_2$.

If $f(x) = \cos x$ then $f(x_1) = f(x_2)$

$\Rightarrow \quad \cos x_1 - \cos x_2 = 0$

$\Rightarrow \quad \sin\frac{x_1 - x_2}{2} \sin\frac{x_1 + x_2}{2} = 0$

$\Rightarrow \quad \sin\frac{x_1 - x_2}{2} = 0$

or $\quad \sin\frac{x_1 + x_2}{2} = 0$

$\Rightarrow \quad x_1 = x_2$ (as $x_1, x_2 \in [0, \pi]$ so $x_1 \neq -x_2$).

If $f(x) = ax + b$ then $f(x_1) = f(x_2)$

$\Rightarrow \quad ax_1 + b = ax_2 + b$

$\Rightarrow \quad a(x_1 - x_2) = 0$

$\Rightarrow \quad x_1 = x_2$

So in any of the above case f is 1 – 1. If $f(x) = e^{x2}$ then $f(1) = f(-1)$. Hence f is not 1 – 1.

Sol.7 (A, B, C)

The function in (A) is periodic with period I. The function in (B) is periodic with period π. The function in (C) on simplification is equal to (1/2) sin 2x, hence is periodic with period π. The function in (D) isnon-periodic.