Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions have only one correct option)

Q.1 Let $f(x) = 1 + \frac{2}{x}$ and $f_n(x) = fff \ldots\ldots f(x)$ then the maximum number of real roots of $f_n(x)$ is/are-

(A) 1 (B) 2
(C) Zero (D) Infinite

Q.2 If [x] and {x} represent integral and fractional part of x then the function defined by

$f(x) = [x] + \sum_{r=1}^{1000} \frac{\{x+r\}}{1000}$ is equal to-

(A) 2[x] + {x} (B) 4x
(C) x (D) 4[x] + 1000 {x}

Q.3 $f(x) = \frac{(x+3)^2}{x^2+1}, -\infty < x < \infty$, then-

(A) $0 \le f(x) \le 3$ (B) $3 \le f(x) \le 9$
(C) $0 \le f(x) \le 10$ (D) $0 \le f(x) \le 7$

Q.4 The set of values of x for which |f(x)| + |g(x)| > |f(x) + g(x)| if f(x) = x – 3 and g(x) = 4 – x

(A) R (B) R – [3, 4]
(C) [3, 4] (D) $(-\infty, 3)$

Q.5 Range of f(x) = [|sin x| + |cos x|], where [.] denotes the greatest integer function is-

(A) {0} (B) {0, 1}
(C) {1} (D) None of these

Q.6 If 1 lies between the roots of equation $y^2 - my + 1 = 0$ and [x] denotes the greatest integer value of x, then value of $\left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]$ is-

(A) –1 (B) 1 (C) 0 (D) $\frac{1}{2}$

Q.7 Solution set of $f^{-1}(x) = x$ is a proper subset of the solution, set of $f(x) = f^{-1}(x)$ then-

(A) Solution set of $f^{-1}(x) = x$ is identical to solution set of f(x) = x
(B) $f(x) = f^{-1}(x)$ has infinite solutions
(C) f(x) & $f^{-1}(x)$ are identical
(D) None of these

Q.8 The range of the function

$\sin(\sin^{-1} x + \cos^{-1} x), |x| \le 1$ is-

(A) [–1, 1] (B) (–1, 1)
(C) {0} (D) {1}

Q.9 If $g(x) = x^2 + x - 2$ and

$\frac{1}{2}(gof)(x) = 2x^2 - 5x + 2$, then f(x) =

(A) 2x – 3 (B) 2x + 3
(C) $2x^2 + 3x + 1$ (D) $2x^2 - 3x - 1$

Q.10 The domain of the function $f(x) = x^{1/\log x}$ is-

(A) $(0, \infty) - \{1\}$ (B) $(0, \infty)$
(C) $[0, \infty)$ (D) $[0, \infty) - \{1\}$

MATHEMATICS IIT JEE (JUNE 3rd WEEK CLASS TEST 3) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	C	C	B	C	C	C	D	A	A

SOLUTIONS

Sol.1 (B)

$f(x) = 1 + \frac{2}{x} = \frac{x+2}{x}$

$\therefore$ Equation $f_n(x)$ is always quadratic in x.

Hence maximum number of real roots are 2.

Sol.2 (C)

Case I : When $x \in I$

$[x] = x$ and $\{x + r\} = 0$ for $r \in I$

Thus $f(x) = x + 0 = x$

Case II : When $x \in R - I$

$[x]$ = integral part of x,

$\{x + r\} = \{x\}$ for r = 1, 2, ... , 1000

Thus $f(x) = [x] + \{x\}$

$= x$

Sol.3 (C)

$f(x) = y = \frac{(x+3)^2}{x^2+1}$, clearly $y \geq 0$ and $(y - 1)x^2 - 6x + y - 9 = 0$ is a quadratic in x and x is real so disc $(\Delta) \geq 0$

$\Rightarrow \quad \Delta = -4y(y - 10)$

$\Rightarrow \quad y \leq 10$ as $y \geq 0$, $\Delta \geq 0$

$\Rightarrow \quad 0 \leq y \leq 10$

Sol.4 (B)

$\therefore \quad f(x) + g(x) = 1$

$\therefore \quad |f(x) + g(x)| = 1$

and $|f(x)| + |g(x)|$

$= 7 - 2x$ if $x < 3$

$= 1$ if $3 \leq x \leq 4$

$= 2x - 7$ if $x > 4$

Now, we want those points for which $|f(x)| + |g(x)| > 1$ clearly for $x < 3$, $7 - 2x > 1$ and for $x > 4$, $2x - 7 > 1$ but for $3 \leq x \leq 4$, $1 \not> 1$.

Thus given inequality holds for $x \in R - [3, 4]$

Sol.5 (C)

Let $\quad y = |\sin x| + |\cos x|$

$\Rightarrow \quad y^2 = 1 + \sin 2x \leq 1 + 1 = 2$

$\Rightarrow \quad 1 \leq y^2 \leq 2$

or $\quad y \in [1, \sqrt{2}]$

Thus $f(x) = [|\sin x| + |\cos x|]$

$= \{1\}$, for $y \in [1, \sqrt{2}]$ & $x \in R$.

Sol.6 (C)

Since 1 lies between the roots of $y^2 - my + 1 = 0$

$\therefore \quad f(1) < 0$

$\Rightarrow \quad 2 - m < 0$

$\Rightarrow \quad m > 2$

We know A.M. $\geq$ G.M.

$\Rightarrow \quad \frac{|x|^2+16}{2} \geq |x|.4$

or $\quad 0 \leq \frac{4.|x|}{|x|^2+16} \leq \frac{1}{2}$

or $\quad 0 \leq \left(\frac{4.|x|}{|x|^2+16}\right)^m < 1$

Hence $\left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right] = 0$

Sol.7 (C)

The curves of $y = f(x)$ and $y = f^{-1}(x)$ will either intersect on line $y = x$ or will coincide.

Given solution set of $f(x) = f^{-1}(x)$ is proper superset of solutions of $f(x) = x$.

$\therefore$ $f(x)$ and $f^{-1}(x)$ must be identical.

Sol.8 (D)

As we know $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$

$\Rightarrow$ $f(x) = \sin \frac{\pi}{2} = 1$

$\therefore$ Range of given function is $\{1\}$

Sol.9 (A)

Given $g(x) = x^2 + x - 2$

and $(gof)(x) = 4x^2 - 10x + 4$

$\Rightarrow$ $(f(x))^2 + f(x) - 2 = 4x^2 - 10x + 4$

$\Rightarrow$ $(f(x))^2 + f(x) - 4x^2 + 10x - 6 = 0$

is quadratic in $f(x)$

$$\Rightarrow f(x) = \frac{-1 \pm \sqrt{1 - 4(-4x^2 + 10x - 6)}}{2}$$

$$= \frac{-1 \pm \sqrt{16x^2 - 40x + 25}}{2}$$

$$= \frac{-1 \pm \sqrt{(4x-5)^2}}{2}$$

$$= \frac{-1 \pm 4x - 5}{2}$$

$$= \frac{4x - 6}{2}, \frac{-4x + 4}{2}$$

$= 2x - 3, -2x + 2$

Sol.10 (A)

$f(x) = x^{\frac{1}{\log x}}$

$f(x)$ is defined for $\log x \neq 0$ and $x > 0$

$\Rightarrow$ $x \neq 10^0$ and $x > 0$

$\Rightarrow$ $x \neq 1$ and $x > 0$

$\Rightarrow$ $x \in (0, \infty) - \{1\}$.