Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (Questions have only one correct option)

Q.1 The function $f : X \to Y$ defined by $f(x) = \sin x$ is one-one but not onto if X and Y are respectively equal to-

(A) IR and IR

(B) $[0, \pi]$ and $[0, 1]$

(C) $[0, \pi/2]$ and $[-1, 1]$

(D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$

Q.2 The value of $n \in I$ for which the function $f(x) = \dfrac{\sin nx}{\sin \frac{x}{n}}$ has 4π as its period is-

(A) 2 (B) 3 (C) 5 (D) 4

Q.3 $f : R \to R$ is a bijection f(x) is a polynomial of degree n. Then-

(A) n must be even

(B) $f'(x) = 0$ cannot have more than 2 solutions

(C) $f'(x) > 0$ for all real x

(D) $f'''(x)$ can be a constant

Q.4 If a polynomial of degree n satisfies $f(x) = f'(x). f''(x) \ \forall \ n \in R$, then f(x) is-

(A) An onto function

(B) An into function

(C) No such function is possible

(D) Even function

Q.5 Let $f : X \to Y$, where $3^{f(x)} + 2^{-x} = 4$ be a one-one and onto function, then-

(A) $X = (-1, \infty), Y = (2, 4)$

(B) $X = (-3, \infty), Y = (0, 4)$

(C) $X = (-2, \infty), Y = (0, 4)$

(D) $X = (-2, \infty), Y = (1, 4)$

Q.6 If f(x) is a polynomial of degree n such that $f(0) = 0, f(1) = \frac{1}{2}, \ldots.. f(n) = \frac{n}{n+1}$, then $f(n + 1) =$

(A) 1, if n is odd (B) –1, if n is odd

(C) $\frac{n}{n-1}$, if n is even (D) $-\frac{n}{n+1}$, if n is even

Q.7 If f(x) and g(x) be periodic and non-periodic functions respectively, then f(g(x)) is-

(A) Always periodic

(B) Never periodic

(C) Periodic, when g(x) is a linear function of 'x'

(D) Can't say

Q.8 The period of $\left[\cos^5\left(\frac{x}{2}\right)\right]$ is-

(A) π (B) 2π (C) 3π (D) None

Q.9 The range of $f(x) = 2\tan^{-1} x + 2\tan^{-1}(x + 1/x - 1)$

(A) $\left\{-\frac{\pi}{2}\right\}$ (B) $\left\{\frac{\pi}{2}\right\}$ (C) $\left\{\frac{3\pi}{2}\right\}$ (D) R

Q.10 Let $f(x) = \int_0^x \log_e\left(\frac{1+x}{1-x}\right)dx$. Then f(x) is-

(A) An even function (B) An odd function

(C) One-one function (D) None of these

MATHEMATICS IIT JEE (JUNE 3rd WEEK CLASS TEST 4) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	D	A	C	A	C	B	C	A

SOLUTIONS

Sol.1 (D)

Function f(x) = sin x such that

f : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow [-1, 1]$ is one-one but not onto because for $-1 \in [-1, 1]$, x = $\frac{3\pi}{2}$, such that $\frac{3\pi}{2} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Sol.2 (A)

For n = 2, we have $\frac{\sin 2x}{\sin x/2} = 4\cos\frac{x}{2}\cos x$

$\because$ period of cos x is 2π & that of

$\cos\frac{x}{2}$ is 4π

Hence period of $\frac{\sin 2x}{\sin\frac{x}{2}}$ is 4π

Also, the period of $\frac{\sin 3x}{\sin\frac{x}{3}}$, $\frac{\sin 4x}{\sin x/4}$ and $\frac{\sin 5x}{\sin x/5}$ .... can not be 4π

Sol.3 (D)

f(x) should be monotonically increasing

let, f(x) = x^3

$\Rightarrow$ f'''(x) = 6 (a constant)

Sol.4 (A)

f(x) = f'(x) . f''(x), $\forall$ x $\in$ R

$\Rightarrow$ degree of fx) is 3(n = n – 1 + n – 2)

$\Rightarrow$ n = 3

$\Rightarrow$ f(x) is onto function.

Sol.5 (C)

$3^{f(x)} = 4 - 2^{-x} \Rightarrow 3^{f(x)}$. In 3. f'(x) = 2^{-x} ln 2

$\Rightarrow$ f'(x) > 0 $\forall$ x $\in$ domain

Domain is $(-2, \infty) \Rightarrow$ Range is (0, 4)

$\Rightarrow$ X = $(-2, \infty)$, Y = (0, 4)

Sol.6 (A)

(x +1) f(x) –x is a polynomial of degree n + 1

$\Rightarrow$ (x + 1) f(x) – x = k(x) [x – 1] [x – 2] .... [x – n]

$\Rightarrow$ [n + 2] f(n + 1) – (n + 1) = k[(n + 1)!]

Also, 1 = k(–1) (–2) ..... ((–n + 1))

1 = $k(-1)^{n+1}$ (n + 1)!

$\Rightarrow$ (n + 2) f(n + 1) – (n + 1) = $(-1)^{n+1}$

$\Rightarrow$ f(n + 1) = 1, if n is odd and $\frac{n}{n+1}$, if n is even.

Sol.7 (C)

Since we know if f(x) is periodic then

f(ax + b) is also periodic function.

Hence g(x) should be a linear function of 'x'.

Sol.8 (B)

Let f(x) = $\left|\cos^5\frac{x}{2}\right| = \left|\cos\frac{x}{2}\right|^5$

$= \left|\sqrt{\cos^2\frac{x}{2}}\right|^5 = \left|\sqrt{\frac{1+\cos x}{2}}\right|^5$

$\because$ Period of cos x is 2π

$\therefore$ Period of f(x) is 2π.

Sol.9 (C)

$f(x) = 2\tan^{-1} x + 2\tan^{-1}(x + 1/x - 1)$

$= 2\tan^{-1} x + 2\tan^{-1}\left(\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}\right)$

$= 2\tan^{-1} x + 2(\tan^{-1} 1 + \tan^{-1} 1/x)$

$= 2(\tan^{-1} x + \cot^{-1} x) + \dfrac{\pi}{2}$

$= 2\left(\dfrac{\pi}{2}\right) + \dfrac{\pi}{2} = \dfrac{3\pi}{2}$

Sol.10 (A)

$f(-x) = \int_0^{-x} \log\left(\dfrac{1+t}{1-t}\right)dt = -\int_0^{x} \log\left(\dfrac{1-u}{1+u}\right)du$

$= \int_0^{x} \log\left(\dfrac{1+x}{1-x}\right)dx = f(x)$

Range f = $[0, \infty)$. Therefore, f is not onto. $f(-1) = f(1)$. Therefore f is not one-one. Since f is not bijective, f^{-1} doesn't exist.