Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-10(+3,–1) (All questions have only one correct option)

Q.1 The period of the function

$$f(x) = \cos\left(\frac{3x}{5}\right) - \sin\left(\frac{2x}{7}\right) \text{ is}$$

(A) 7π (B) 10π (C) 70π (D) 35π

Q.2 The graph of the function
$f(x) = \cos x \cos (x + 2) - \cos^2 (x + 1)$
is
(A) A straight line through $(0, -\sin^2 1)$ with slope 2
(B) A straight line through (0, 0)
(C) A parabola with vertex at $(1, -\sin^2 1)$
(D) A straight line through $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to x-axis.

Q.3 If $f(x) = x^n$, $n \in N$ and (gof) (x) = ng(x), then g(x) can be
(A) $n\ |x|$ (B) $3\sqrt[3]{x}$
(C) e^x (D) $\log |x|$

Q.4 If f(1) = 1, f(x) is defined for $1 \le x < \infty$ and

$$f'(x) = \frac{1}{x^2 + (f(x))^2}$$, then f(x) is less than

(A) $1 - \frac{\pi}{4}$ (B) $1 + \frac{\pi}{4}$
(C) $\frac{\pi}{4}$ (D) None

Q.5 Let $f : R \to R$ defined by
$f(x) = x^3 + x^2 + 100x + 5 \sin x$, then f is
(A) one one into (B) one one onto
(C) many one into (D) many one onto

Q.6 If f(x) and g(x) are two functions of x such that $f(x) + g(x) = e^x$ and $f(x) - g(x) = e^{-x}$. Then
(A) f(x) is odd, g(x) is odd
(B) f(x) is even, g(x) is even
(C) f(x) is even, g(x) is odd
(D) f(x) is odd, g(x) is even

Q.7 If the function $f : R \to R$ be such that $f(x) = x - [x]$ where [x] denotes the greater integer function less, then or equal to x, then $f^{-1}(x)$ is
(A) $\frac{1}{x - [x]}$ (B) $[x] - x$
(C) not defined (D) $[x] + x$

Q.8 A function y = f(x) is periodic with period T, then which of the following is true :
(A) It is one-one in (0, T)
(B) It is many one in (0, T)
(C) It may or may not be one-one in (0, T)
(D) None

Q.9 Let g(x) be a function defined on [–1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and (x, g(x)) is $\frac{\sqrt{3}}{4}$, then the function g(x) is
(A) $g(x) = \sqrt{1 - x^2}$ (B) $\pm\sqrt{1 - x^2}$
(C) $-\sqrt{1 - x^2}$ (D) $\sqrt{1 + x^2}$

Q.10 If $f : R \to S$ defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$ is onto, then the interval of S is
(A) [–1, 3] (B) [–1, 1]
(C) [0, 1] (D) [0, 3]

MATHEMATICS IIT JEE (JUNE 3rd WEEK CLASS TEST 5) (FUNCTIONS) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	D	D	B	B	C	C	C	B	A

SOLUTIONS

Sol.1 (C)

Period of $\cos\frac{3x}{5} = \frac{2\pi}{3/5} = \frac{10\pi}{3}$

Period of $\sin\frac{2x}{7} = \frac{2\pi}{2/7} = \frac{14\pi}{2} = \frac{7\pi}{1}$

$\therefore$ Period of $f(x) = \frac{\pi(\text{L.C.M. of 10 and 7})}{(\text{H.C.F. of 3 and 1})}$

$= \frac{70\pi}{1} = 70\pi$

Sol.2 (D)

$y = f(x) = \cos x \cos(x+2) - \cos^2(x+1)$

$= \frac{1}{2}[\cos 2(x+1) + \cos 2] - \cos^2(x+1)$

$= \frac{1}{2}[2\cos^2(x+1) - 1 + \cos 2] - \cos^2(x+1)$

$= \frac{-1+\cos 2}{2} = \frac{-1+1-2\sin^2 1}{2} = -\sin^2 1$

Thus graph of the function is a st. line parallel to x-axis and passing through $\left(\frac{\pi}{2}, -\sin^2 1\right)$.

Sol.3 (D)

$\because$ $gof(x) = g(x^n)$

Given $gof(x) = ng(x)$

$\Rightarrow$ $g(x^n) = ng(x)$ is possible

if $g(x) = \log|x|$.

Sol.4 (B)

Given $f'(x) = \frac{1}{x^2+(f(x)^2)} > 0$

As f(x) is increasing for $1 \le x < \infty$

$f(x) > f(1) = 1$

$\therefore$ $x^2 + (f(x))^2 \ge x^2 + 1$

$\Rightarrow$ $\frac{1}{x^2+(f(x))^2} \le \frac{1}{x^2+1}$

$\therefore$ $\int_1^x f'(x)\,dx = \int_1^x \frac{1}{x^2+(f(x))^2}\,dx \le \int_1^x \frac{1}{x^2+1}\,dx$

$\le \int_1^\infty \frac{1}{1+x^2}\,dx$

$\Rightarrow$ $f(x) - f(1) < \frac{\pi}{4}$

$\Rightarrow$ $f(x) < 1 + \frac{\pi}{4}$ $[\because f(1) = 1]$

Sol.5 (B)

$f(x) = x^3 + x^2 + 100x + 5\sin x$

$f'(x) = 3x^2 + 2x + 100 + 5\cos x$

$= 3\left(x+\frac{1}{3}\right)^2 + \left(94 - \frac{1}{3}\right) + (6 + 5\cos x) > 0$

$\Rightarrow$ f is an increasing function.

$\Rightarrow$ f is one one function.

Also $f(-\infty) = -\infty$ and $f(\infty) = \infty$

$\Rightarrow$ f is continuous

$\therefore$ Range of f = Co-domain of f.

$\Rightarrow$ f is onto.

Sol.6 (C)

Given $f(x) + g(x) = e^x$

$f(x) - g(x) = e^{-x}$

$\Rightarrow$ $f(x) = \frac{e^x + e^{-x}}{2}$ and $g(x) = \frac{e^x - e^{-x}}{2}$

Clearly $f(-x) = f(x)$ and $g(-x)$ and $-g(x)$

$\Rightarrow$ f(x) is even and g(x) is odd function.

Sol.7 (C)

Given $f(x) = x - [x]$

Now $f(1) = 1 - (1) = 0$

and $f(0) = 0$

But $0 \ne 1$

$\Rightarrow$ f is not one-one.

$\Rightarrow$ f^{-1} is not defined.

Sol.8 (C)

Let us take example

$f(x) = \{x\}$ is one one in their respective period where as $f(x) = \sin x$ is many one in their respective period.

Sol.9 (B)

Given area of the equilateral triangle = $\frac{\sqrt{3}}{4}$

and sides of equilateral triangle = $\sqrt{(x-0)^2 + (g(x)-0)^2} = \lambda$ say

$\Rightarrow \quad \lambda^2 = x^2 + (g(x))^2$

$\therefore \quad \frac{\sqrt{3}}{4}\lambda^2 = \frac{\sqrt{3}}{4}$

$\Rightarrow \quad \lambda^2 = 1 \qquad \Rightarrow \qquad \lambda = 1$

$\therefore \quad x^2 + (g(x))^2 = 1$

or $\quad g(x) = \pm\sqrt{1-x^2}$

Sol.10 (A)

Given f(x) is onto, so S = range of f(x)

$\because \quad f(x) = \sin x - \sqrt{3}\cos x + 1$

$= 2\sin\left(x - \frac{\pi}{3}\right) + 1$

As $\quad -1 \le \sin x \le 1$

$\therefore \quad -1 \le \sin\left(x - \frac{\pi}{3}\right) \le 1$

$-2 \le 2\sin\left(x - \frac{\pi}{3}\right) \le 2$

$-1 \le 2\sin\left(x - \frac{\pi}{3}\right) + 1 \le 3$

$\Rightarrow \quad f(x) \in [-1, 3] = S$