Dear student following is an Easy level [● O O] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-7(+3, –1) (Questions may have more than option correct)

Q.1 $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}}\,dx =$

(A) $\ell n\left(x+\sqrt{x^2-1}\right) - \sec^{-1} x + c$

(B) $\ell n\left(x+\sqrt{x^2-1}\right) + \sec^{-1} x + c$

(C) $\ell n\left(x-\sqrt{x^2-1}\right) - \sec^{-1} x + c$

(D) $\ell n\left(x-\sqrt{x^2-1}\right) + \sec^{-1} x + c$

Q.2 If $I = \int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx = f(x) + c$ then f(x) is equal to-

(A) $\sqrt{2}\ \sin^{-1}(\sin x - \cos x)$

(B) $\frac{\pi}{\sqrt{2}} - \sqrt{2}\ \cos^{-1}(\sin x - \cos x)$

(C) $\sqrt{2}\ \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2}\sqrt{\tan x}}\right)$

(D) None of these

Q.3 If $\sqrt{\frac{\cos x - \cos^3 x}{(1-\cos^3 x)}}\,dx = f(x) + c$, then f(x) is equal to-

(A) $\frac{2}{3}\sin^{-1}\left(\cos^{3/2} x\right)$ (B) $\frac{3}{2}\sin^{-1}(\cos^{3/2}x)$

(C) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x)$ (B) $-\frac{2}{3}\sin^{-1}(\cos^{3/2}x)$

Q.4 $\int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{(x-x^2)}}$ is equal to-

(A) $\frac{1+\sqrt{x}}{(1-x)^2} + c$ (B) $\frac{1+\sqrt{x}}{(1+x)^2} + c$

(C) $\frac{1-\sqrt{x}}{(1-x)^2} + c$ (D) $\frac{2\left(\sqrt{x}-1\right)}{\sqrt{(1-x)}} + c$

Q.5 $\int \frac{x^4-1}{x^2(x^4+x^2+1)^{1/2}}\,dx$ is equal to-

(A) $\sqrt{\frac{x^4+x^2+1}{x}} + c$ (B) $\sqrt{x^2+1+\frac{1}{x^2}} + c$

(C) $\sqrt{\frac{x^4+x^2+1}{x^2}} + c$ (D) None of these

Q.6 $\int \frac{3+2\cos x}{(2+3\cos x)^2}\,dx$ is equal to-

(A) $\left(\frac{\sin x}{3\cos x+2}\right) + c$ (B) $\left(\frac{2\cos x}{3\sin x+2}\right) + c$

(C) $\left(\frac{2\cos x}{3\cos x+2}\right) + c$ (D) $\left(\frac{2\sin x}{3\sin x+2}\right) + c$

Q.7 $\int \frac{(x^2-1)}{(x^2+1)\sqrt{x^4+1}}\,dx$ is equal to-

(A) $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$

(B) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$

(C) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$

(D) None of these

MATHEMATICS IIT JEE (SEPT. 1st WEEK CLASS TEST 1) (INDEFINITE INTEGRATION) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○					
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	A	A,B,C	C,D	D	B,C	A	B

SOLUTIONS

Sol.1 (A)

$$\text{Let } I = \int \frac{\sqrt{x-1}}{x\sqrt{x+1}}\,dx = \int \frac{x-1}{x\sqrt{x^2-1}}\,dx$$

$$= \int \frac{1}{\sqrt{x^2-1}}\,dx - \int \frac{dx}{x\sqrt{x^2-1}}$$

$$= \ell n\left(x+\sqrt{x^2-1}\right) - \sec^{-1} x + c$$

Sol.2 (A, B, C)

$$I = \int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$= \int \sqrt{2}\frac{\sin x+\cos x}{\sqrt{2\sin x\cos x}}\,dx \text{ If } \sin x - \cos x = p$$

then $(\cos x + \sin x)\,dx = dp$

$$\Rightarrow \quad I = \sqrt{2}\int \frac{dp}{\sqrt{1-p^2}} = \sqrt{2}\ \sin^{-1} p + c$$

$$= \sqrt{2}\ \sin^{-1}(\sin x - \cos x) + c$$

$$= \frac{\pi}{\sqrt{2}} - \sqrt{2}\ \cos^{-1}(\sin x - \cos x)$$

$$= \sqrt{2}\ \tan^{-1}\frac{\sin x-\cos x}{\sqrt{1-(\sin x-\cos x)^2}}$$

$$= \sqrt{2}\ \tan^{-1}\frac{\sin x-\cos x}{\sqrt{2\sin x\cos x}}$$

$$= \sqrt{2}\ \tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)$$

Sol.3 (C, D)

$$\text{Let } I = \int \sqrt{\frac{\cos x-\cos^3 x}{(1-\cos^3 x)}}\,dx$$

$$= \int \frac{\sqrt{\cos x}\left(\sqrt{1-\cos^2 x}\right)}{\sqrt{1-\left(\cos^{3/2} x\right)^2}}\,dx$$

If $\cos^{3/2} x = p$, then $\left(-\frac{3}{2}\cos^{1/2} x\sin x\right)dx = dp$

$$I = -\frac{2}{3}\int \frac{dp}{\sqrt{1-p^2}} = \frac{2}{3}\sin^{-1}(\cos^{3/2} x) + c$$

$$= \frac{2}{3}\ \cos^{-1}(\cos^{3/2} x) + C_1$$

sol.4 (D)

$$\text{Let } I = \int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{(x-x^2)}}$$

If $\sqrt{x} = \sin p$, then $\frac{1}{2\sqrt{x}}dx = \cos p\ dp$

$$I = \int \frac{2\sin p\cos p\,dp}{(1+\sin p)\sin p\cos p} = 2\int \frac{dp}{(1+\sin p)}$$

$$= 2\int \frac{(1-\sin p)\,dp}{\cos^2 p}$$

$$= 2\left\{\int \sec^2 p\,dp - \int (\tan p\sec p)\,dp\right\}$$

$$= 2\ (\tan p - \sec p)$$

$$= 2\left(\sqrt{\frac{x}{(1-x)}} - \frac{1}{\sqrt{(1-x)}}\right) + c$$

$$= \frac{2\left(\sqrt{x}-1\right)}{\sqrt{(1-x)}} + c$$

Sol.5 (B, C)

$$\text{Let } I = \int \frac{x^4 - 1}{x^2 (x^4 + x^2 + 1)^{1/2}} \, dx$$

$$= \int \frac{x^3 \left(x - \frac{1}{x^3} \right) dx}{x^3 \sqrt{x^2 + \frac{1}{x^2} + 1}}$$

$$\text{If } x^2 + \frac{1}{x^2} + 1 = p^2,$$

$$\text{then } \left(2x - \frac{2}{x^3} \right) dx = 2p \, dp$$

$$\Rightarrow I = \int \frac{p \, dp}{p} = p + c = \sqrt{\frac{x^4 + x^2 + 1}{x^2}} + c$$

Sol.6 (A)

$$\text{Let } I = \int \frac{3 + 2\cos x}{(2 + 3\cos x)^2} \, dx$$

Multiplying Nr. & Dr. by $\text{cosec}^2 x$

$$\Rightarrow I = \int \frac{(3\,\text{cosec}^2 x + 2\cot x \,\text{cosec}\, x)}{(2\,\text{cosec}\, x + 3\cot x)^2} \, dx$$

$$= - \int \frac{-3\,\text{cosec}^2 x - 2\cot x \,\text{cosec}\, x}{(2\,\text{cosec}\, x + 3\cot x)^2} \, dx$$

$$= \frac{1}{2\,\text{cosec}\, x + 3\cot x} = \left(\frac{\sin x}{2 + 3\cos x} \right) + c$$

Sol.7 (B)

$$I = \int \frac{x^2 \left(1 - \frac{1}{x^2} \right) dx}{x^2 \left(x + \frac{1}{x} \right) \left(x^2 + \frac{1}{x^2} \right)^{1/2}}$$

$$\text{Let } x + \frac{1}{x} = p \Rightarrow \left(1 - \frac{1}{x^2} \right) dx = dp$$

$$I = \int \frac{dp}{p \sqrt{p^2 - 2}} = \frac{1}{\sqrt{2}} \sec^{-1} \frac{p}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \sec^{-1} \left(\frac{x^2 + 1}{\sqrt{2} x} \right) + c$$