Dear student following is a Moderate level [○●○] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-7(+3, –1) (Questions may have more than option correct)

Q.1 $I = \int \frac{(x + x^{2/3} + x^{1/6})}{x(1 + x^{1/3})}\,dx$ is equal to-

(A) $\frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$

(B) $\frac{3}{2}x^{2/3} - 6\tan^{-1}(x^{1/6}) + c$

(C) $\frac{3}{2}x^{2/3} + \tan^{-1}(x^{1/6}) + c$

(D) None of these

Q.2 $\int \left(\frac{\ell n\, x - 1}{(\ell n\, x)^2 + 1}\right)^2 dx$ is equal to-

(A) $\frac{x}{x^2+1} + c$ (B) $\frac{\ell n x}{(\ell n x)^2 + 1}$

(C) $\frac{x}{(\ell n x)^2+1} + c$ (D) $e^x\left(\frac{x}{x^2+1}\right) + c$

Q.3 If $\int \frac{x + (\cos^{-1}3x)^2}{\sqrt{1-9x^2}}\,dx = A\sqrt{1-9x^2} + B\left(\cos^{-1}3x\right)^3 + c$, where c is integration constant, then the value of A and B are-

(A) $A = -\frac{1}{9}$; $B = -\frac{1}{9}$

(B) $A = -\frac{1}{9}$; $B = \frac{1}{9}$

(C) $A = \frac{1}{9}$; $B = \frac{1}{9}$

(D) None of these

Q.4 If $\int \sec 2x\,dx = f[g(x)] + C$, then

(A) dom $f(x) = R - \{0\}$

(B) Range of $g(x) = R$

(C) $f'(x) = \frac{1}{2x}$, $\forall\, x \in R^+$

(D) $g'(x) = -\operatorname{cosec}^2\left(\frac{\pi}{4} - x\right)$

Q.5 Let $f(x) = \frac{x+1}{x+2}$. If $\int \left(\frac{f(x)}{x^2}\right)^{1/2} dx = \frac{1}{\sqrt{2}}\, g\left(\frac{\sqrt{2f(x)} - 1}{\sqrt{2f(x)} + 1}\right) - h\left(\frac{\sqrt{f(x)} - 1}{\sqrt{f(x)} + 1}\right)$, then

(A) $g(x) = \log|x|$, $h(x) = \log|x|$

(B) $g(x) = \log|x|$, $h(x) = \tan^{-1} x$

(C) $g(x) = \tan^{-1} x$, $h(x) = \log|x|$

(D) None of these

Q.6 If $\int \frac{2^{1/x}}{x^2}\,dx = K.2^{1/x}$, then K is equal to

(A) $\frac{-1}{\log 2}$ (B) $-\log 2$

(C) -1 (D) $\frac{1}{2}$

Q.7 The value of $\int \frac{(x - x^3)^{1/3}}{x^4}\,dx$ is-

(A) $\frac{1}{8}\left(1 - \frac{1}{x^2}\right)^{4/3} + 1 + C$

(B) $\frac{3}{8}\left(\frac{1}{x^2} - 1\right)^{4/3} + C$

(C) $\frac{-3}{8}\left(\frac{1}{x^2} - 1\right)^{4/3} + C$

(D) None of these

MATHEMATICS IIT JEE (SEPT. 2nd WEEK CLASS TEST 1) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭					
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	A	C	A	A,B,C	A	A	C

SOLUTIONS

Sol.1 (A)

Substituting $x = p^6$, $dx = 6p^5\,dp$, we have

$$I = \int \frac{6p^5(p^6+p^4+p)}{p^6(1+p^2)}\,dp$$

$$= \int \frac{6(p^5+p^3+1)}{(p^2+1)}\,dp$$

$$= \int 6p^3\,dp + \int \left(\frac{6}{p^2+1}\right)dp$$

$$= \frac{6p^4}{4} + 6\tan^{-1} p$$

$$= \frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$$

Sol.2 (C)

Put $\ell n\, x = t$

$$I = \int e^t\left(\frac{t-1}{t^2+1}\right)^2 dt$$

$$= \int e^t\left(\frac{1}{t^2+1} - \frac{2t}{(t^2+1)^2}\right) dt$$

$$= \frac{e^t}{t^2+1} + c = \frac{x}{(\ell n x)^2+1} + c.$$

Sol.3 (A)

Let $3x = \cos\theta \Rightarrow 3\,dx = -\sin\theta\,d\theta$

$$\therefore\quad -\frac{1}{3}\int \frac{\frac{\cos\theta}{3}+\theta^2}{\sin\theta}\sin\theta\,d\theta$$

$$= -\frac{1}{3}\int\left(\frac{1}{3}\cos\theta+\theta^2\right)d\theta$$

$$= -\frac{1}{9}\sin\theta - \frac{1}{9}\theta^3 + c$$

$$= -\frac{1}{9}\sqrt{1-9x^2} - \frac{1}{9}(\cos^{-1}3x)^3 + c$$

Sol.4 (A, B, C)

$$\int \sec 2x\,dx = \frac{1}{2}\int \frac{2\sec 2x(\sec 2x+\tan 2x)}{(\sec 2x+\tan 2x)}\,dx$$

$$= \frac{1}{2}\log|\sec 2x + \tan 2x| + C$$

$$= \frac{1}{2}\log\left|\frac{1+\sin 2x}{\cos 2x}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{1+\cos\left(\frac{\pi}{2}-2x\right)}{\sin\left(\frac{\pi}{2}-2x\right)}\right| + C$$

$$= \frac{1}{2}\log\left|\cot\left(\frac{\pi}{4}-x\right)\right| + C$$

$\therefore f(X) = \frac{1}{2}\log|x|$ and $g(x) = \cot\left(\frac{\pi}{4}-x\right)$

$\therefore$ dom $f(x) = (-\infty, \infty) - \{0\}$
and range $g(x) = (-\infty, \infty)$

Also, $f'(x) = \frac{1}{2x}$, $\forall\ x \in R^+$

and $g'(x) = \text{cosec}^2\left(\frac{\pi}{4}-x\right)$

Sol.5 (A)

Put $f(x) = y^2 = \frac{x+1}{x+2}$

$$\Rightarrow\quad x = \frac{2y^2-1}{1-y^2} \text{ and } dx = \frac{2y}{(1-y^2)^2}\,dy$$

$\therefore \quad \int \left(\frac{f(x)}{x^2}\right)^{1/2} dx$

$= \int y.\frac{1-y^2}{2y^2-1} \cdot \frac{2y}{(1-y^2)^2} dy$

$= 2\int \frac{y^2}{(2y^2-1)(1-y^2)} dy$

$= 2 \int \left[\frac{1}{2y^2-1} - \frac{1}{y^2-1}\right] dy$

$= \frac{1}{\sqrt{2}} \log \left|\frac{\sqrt{2}y-1}{\sqrt{2}y+1}\right| - \log \left|\frac{y-1}{y+1}\right|$

$= \frac{1}{\sqrt{2}} \log \left|\frac{\sqrt{2f(x)}-1}{\sqrt{2f(x)}+1}\right| - \log \left|\frac{\sqrt{f(x)}-1}{\sqrt{f(x)}+1}\right|$

Thus, $g(x) = \log |x|$ and $h(x) = \log |x|$.

Sol.6 (A)

We have, $\int \frac{2^{1/x}}{x^2} dx = K.2^{1/x}$

Differentiating both sides w.r.t. x, we get

$\frac{2^{1/x}}{x^2} = K.2^{1/x} . \left(\frac{-1}{x^2}\right) . \log 2$

$\Rightarrow \quad K = \frac{-1}{\log 2}$

Sol.7 (C)

$\int \frac{(x-x^3)^{1/3}}{x^4} dx = \int \frac{1}{x^3}\left(\frac{1}{x^2}-1\right)^{1/3} dx$

$= \frac{-1}{2} \int t^{1/3} dt$

$\left[\text{Putting } \frac{1}{x^2}-1 = t \Rightarrow \frac{-2}{x^3}dx = dt\right]$

$= \frac{-1}{2} . \frac{t^{4/3}}{4/3} + C = \frac{-3}{8} \left(\frac{1}{x^2}-1\right)^{4/3} + C$