Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 24 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (Questions may have more than one option correct).

Q.1 Locus of z if

$$\arg[z-(1+i)] = \begin{cases} \frac{3\pi}{4} & \text{when } |z| \le |z-2| \\ \frac{-\pi}{4} & \text{when } |z| > |z-2| \end{cases}$$ is-

(A) Straight lines passing through (2, 0)
(B) Straight lines passing through (2, 0), (1, 1)
(C) A line segment
(D) A set of two rays

Q.2 Let $z \in C$ and if $A = \left\{z \; \arg(z) = \frac{\pi}{4}\right\}$ and $B = \left\{z \; \arg(z-3-3i) = \frac{2\pi}{3}\right\}$. Then $n(A \cap B)$ is equal to-

(A) 1 (B) 2 (C) 3 (D) 0

Q.3 If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are complex numbers such that $|z_1| = 1$, $|z_2| = 2$ and $\text{Re}(z_1z_2) = 0$, then the pair of complex numbers $\omega_1 = a_1 + \frac{ia_2}{2}$ and $\omega_2 = 2b_1 + ib_2$ satisfy-

(A) $|\omega_1| = 1$ (B) $|\omega_2| = 2$
(C) $\text{Re}(\omega_1\omega_2) = 0$ (D) $\text{Im}(\omega_1\omega_2) = 2$

Q.4 Let z_k (k = 0, 1, 2, ... , 6) be the roots of the equation $(z + 1)^7 + z^7 = 0$, then $\sum_{k=0}^{6} \text{Re}(z_k)$ is equal to-

(A) 0 (B) $\frac{3}{2}$ (C) $-\frac{7}{2}$ (D) $\frac{7}{2}$

Q.5 The cube roots of unity-
(A) Lie on the circle |z| = 1
(B) Are collinear
(C) Form an equilateral triangle
(D) None of these

Q.6 If z_1 and z_2 are complex numbers, such that $z_1 + z_2$ is a real number, then-
(A) $z_1 = -\bar{z}_2$ (B) $z_2 = \bar{z}_1$
(C) z_1 and z_2 are any two complex numbers
(D) $z_1 = \bar{z}_1$, $z_2 = \bar{z}_2$

Q.7 If z_1, z_2, z_3 are three complex numbers in A.P., then they lie on-
(A) A circle (B) An ellipse
(C) A straight line (D) A parabola

Q.8 Let z_1 and z_2 be nth roots of unity which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form-
(A) 4k + 1 (B) 4k + 2
(C) 4k + 3 (D) 4k

Q.9 For any integer n, the argument of

$$z = \frac{\left(\sqrt{3}+i\right)^{4n+1}}{\left(1-i\sqrt{3}\right)^{4n}}$$ is-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

Q.10 The closest distance of the origin from a curve given as $a\bar{z} + \bar{a}z + a\bar{a} = 0$ (a is a complex number) is-

(A) 1 (B) $\frac{|a|}{2}$ (C) $\frac{\text{Re}\, a}{|a|}$ (D) $\frac{\text{Im}\, a}{|a|}$

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 1) (COMPLEX NUMBER) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	D	All	C	A,C	B	C	D	A	B

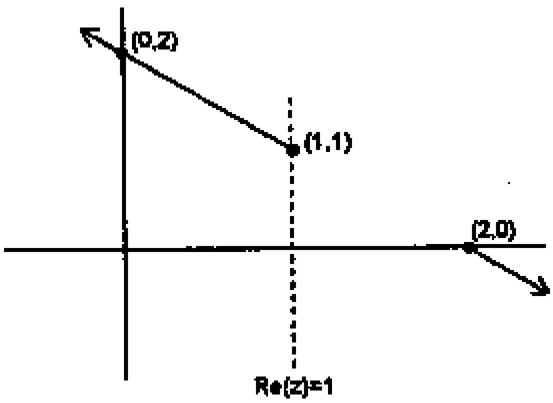
SOLUTIONS

Sol.1 **(D)**

The given equation is written as

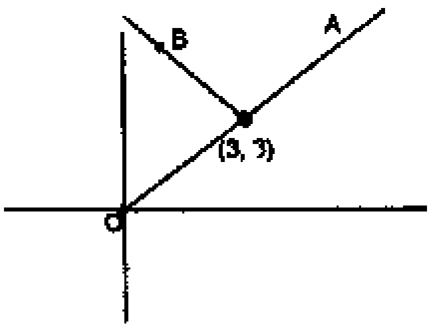
$$\arg(z - (1 + i)) = \begin{cases} \frac{3\pi}{4} & \text{when } x \le 2 \\ \frac{-\pi}{4} & \text{when } x > 2 \end{cases}$$

$\Rightarrow$ the locus is a set of two rays.



Sol.2 **(D)**

We can observe that $3 + 3i \in A$ but $\notin B$



$\therefore\ n(A \cap B) = 0$

Sol.3 **(All)**

$a_1^2 + b_1^2 = 1$

$a_2^2 + b_2^2 = 4$ and $a_1a_2 = b_1b_2$

$a_2^2 + b_2^2 = 4a_1^2 + 4b_1^2$

$(a_2 + 2ia_1)^2 = (2b_1 + ib_2)^2$

$\Rightarrow\ a_2 = \pm 2b_1$

$2a_1 = \pm b_2$

$$|\omega_1|^2 = a_1 + a_1^2 + \frac{a_2^2}{4} - a_1^2 + b_1^2 = 1$$

$\Rightarrow\ |\omega_1| = 1$

and $|\omega_2|^2 = 4b_1^2 + b_2^2 = 4$

$\Rightarrow\ |\omega_2| = 2$

$$\text{Re}(\omega_1\omega_2) = 2a_1b_1 - \frac{a_2b_2}{2} = 0$$

$$\text{Im}(\omega_1\omega_2) = a_1b_2 + a_2b_1 = 2a_1^2 + 2b_1^2 = 2$$

Sol.4 **(C)**

Let $z_k = x_k + iy_k$,

we have $(z_k + 1)^7 + z_k^7 = 0$

$\Rightarrow\ (z_k + 1)^7 = -z_k^7$

$\Rightarrow\ |z_k + 1|^7 = |z_k|^7$

$\Rightarrow\ |z_k + 1| = |z_k|$

$\Rightarrow\ |x_k + iy_k + 1|^2 = |x_k + iy_k|^2$

$\Rightarrow\ (x_k + 1)^2 + y_k^2 = x_k^2 + y_k^2$

$\Rightarrow\ 2x_k + 1 = 0$ or $x_k = -\frac{1}{2}$

Thus, $\sum_{k=0}^{6}\text{Re}(z_k) = \sum_{k=0}^{6}x_k = -\frac{7}{2}$.

Sol.5 **(A, C)**

Clearly, cube roots of unity $1, \omega, \omega^2$ satisfy $|z| = 1$.

Also, $|1 - \omega|^2 = \left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 3$

$\Rightarrow\ |1 - \omega| = \sqrt{3}$

$|\omega - \omega^2| = |\sqrt{3}\,i| = \sqrt{3}$

and $|1 - \omega^2| = \left|1 - \left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\right|$

$= \left|\frac{3}{2} + \frac{i\sqrt{3}}{2}\right| = \sqrt{3}$

Therefore, $1, \omega, \omega^2$ form an equilateral triangle.

Sol.6 **(B)**

Let $z_1 = x_1 + iy_1$

$\therefore\ \bar{z}_1 = x_1 - iy_1$

Now $z_2 = \bar{z}_1$

$\Rightarrow\ z_1 + z_2 = z_1 + \bar{z}_1$

$\Rightarrow\ x_1 + iy_1 + x_1 - iy_1 = 2x_1$, which is real.

Hence result holds if $z_2 = \bar{z}_1$.

Sol.7 (C)

$\because$ z_1, z_2, z_3 are in A.P.

$\therefore$ $z_1 + z_3 = 2z_2$ or $z_2 = \dfrac{z_1 + z_3}{2}$

$\therefore$ z_2 is the mid point of the join of z_1 and z_3. Hence they lie on a straight line.

Sol.8 (D)

We have, $1^{1/n} = \cos\dfrac{2r\pi}{n} + i\sin\dfrac{2r\pi}{n}$

Let $z_1 = \cos\dfrac{2r_1\pi}{n} + i\sin\dfrac{2r_1\pi}{n}$

and $z_2 = \cos\dfrac{2r_2\pi}{n} + i\sin\dfrac{2r_2\pi}{n}$

Then $\angle Z_1OZ_2 = \text{amp}\,\dfrac{z_1}{z_2}$

$= \text{amp}\, z_1 - \text{amp}\, z_2$

$= \dfrac{2(r_1 - r_2)\pi}{n} = \dfrac{\pi}{2}$ (given).

$\therefore$ $n = 4(r_1 - r_2) = 4 \times$ integer.

So, n is of the form 4k.

Sol.9 (A)

We have, $z = \dfrac{(\sqrt{3}+i)^{4n+1}}{(1-i\sqrt{3})^{4n}}$

$$= \frac{(2e^{i\pi/6})^{4n+1}}{(2e^{-i\pi/3})^{4n}} = \frac{2^{4n+1}\, e^{i(4n+1)\frac{\pi}{6}}}{2^{4n}\, e^{-i4n\frac{\pi}{3}}}$$

$$= 2.e^{i(12n+1)\frac{\pi}{6}} = 2.e^{2n\pi i}\,.\,e^{\frac{\pi i}{6}}$$

$$= 2.e^{\pi i/6} \qquad (\because\ e^{2n\pi i} = 1)$$

$\therefore$ $\arg z = \dfrac{\pi}{6}$

Sol.10 (B)

The closest distance = length of the perpendicular from the origin on the line $a\bar{z} + \bar{a}z + a\bar{a} = 0$

$$= \frac{a(0) + \bar{a}|0| + a\bar{a}}{2|a|} = \frac{|a|^2}{2|a|} = \frac{|a|}{2}$$