Dear student following is a Moderate level [○●○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3, –1) (All questions have only one option correct)

Q.1 $\int \frac{\sec x dx}{\sqrt{\cos 2x}} =$

(A) $\sin^{-1}(\tan x)$ (B) $\tan x$

(C) $\cos^{-1}(\tan x)$ (D) $\frac{\sin x}{\sqrt{\cos x}}$

Q.2 $\int \text{tain}(3x - 5) \sec(3x - 5)\, dx =$

(A) $\sec(3x - 5) + c$

(B) $\frac{1}{3}\sec(3x - 5) + c$

(C) $\tan(3x - 5) + c$ (D) None of these

Q.3 If $\int \frac{1}{x + x^5} dx = f(x) + c$, then the value of $\int \frac{x^4}{x + x^5} dx$ is-

(A) $\log x - f(x) + c$ (B) $f(x) + \log x + c$

(C) $f(x) - \log x + c$ (D) None of these

Q.4 If $x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, then $\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x\, dx =$

(A) $e^{\sin x} + c$ (B) $e^{\sin x - \cos x} + c$

(C) $e^{\sin x + \cos x} + c$ (D) $e^{\cos x - \sin x} + c$

Q.5 $\int \frac{x^2}{(9 - x^2)^{3/2}} dx =$

(A) $\frac{x}{\sqrt{9 - x^2}} = \sin^{-1}\frac{x}{3} + c$

(B) $\frac{x}{\sqrt{9 - x^2}} + \sin^{-1}\frac{x}{3} + c$

(C) $\sin^{-1}\frac{x}{3} - \frac{x}{\sqrt{9 - x^2}} + c$ (D) None of these

Q.6 $\int \frac{dx}{(2\sin x + \cos x)^2} =$

(A) $\frac{1}{2}\left(\frac{1}{2\tan x + 1}\right) + c$

(B) $\frac{1}{2}\log(2\tan x + 1) + c$

(C) $\frac{1}{2 + \cot x} + c$ (D) $-\frac{1}{2}\left(\frac{1}{2\tan x - 1}\right) + c$

Q.7 If $\int \frac{1}{(\sin x + 4)(\sin x - 1)} dx = A\frac{1}{\tan x/2 - 1} + B\tan^{-1}(f(x)) + C$, then-

(A) $A = \frac{1}{5}$, $B = \frac{-2}{5\sqrt{15}}$, $f(x) = \frac{4\tan x + 3}{\sqrt{15}}$

(B) $A = -\frac{1}{5}$, $B = \frac{1}{\sqrt{15}}$, $f(x) = \frac{4\tan\left(\frac{x}{2}\right) + 1}{\sqrt{15}}$

(C) $A = \frac{2}{5}$, $B = \frac{-2}{5}$, $f(x) = \frac{4\tan x + 1}{5}$

(D) $A = \frac{2}{5}$, $B = \frac{-2}{5\sqrt{15}}$, $f(x) = \frac{4\tan\frac{x}{2} + 1}{\sqrt{15}}$

Q.8 $\int \frac{1}{x^2}\log(x^2 + a^2)\, dx =$

(A) $\frac{1}{x}\log(x^2 + a^2) + \frac{2}{a}\tan^{-1}\frac{x}{a} + c$

(B) $-\frac{1}{x}\log(x^2 + a^2) + \frac{2}{a}\tan^{-1}\frac{x}{a} + c$

(C) $-\frac{1}{x}\log(x^2 + a^2) - \frac{2}{a}\tan^{-1}\frac{x}{a} + c$

(D) None of these

MATHEMATICS IIT JEE (SEPT. 2nd WEEK CLASS TEST 2) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	B	A	A	A	C	D	B

SOLUTIONS

Sol.1 (A)

$$\int \frac{\sec x\,dx}{\sqrt{\cos 2x}} = \int \frac{\sec x}{\sqrt{\cos^2 x - \sin^2 x}}\,dx$$

$$= \int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}}$$ {Multiplying N'r and D'r by sec x}

Now putting tan x = t $\Rightarrow$ $\sec^2 x\,dx = dt$, we get the integral = $\sin^{-1} t = \sin^{-1}(\tan x)$

Sol.2 (B)

Put t = 3x − 5 $\Rightarrow$ dt = 3 dx, therefore

$$\int \tan(3x-5)\sec(3x-5)\,dx$$

$$= \frac{1}{3}\int \tan t.\sec t\,dt$$

$$= \frac{\sec t}{3} + c = \frac{\sec(3x-5)}{3} + c$$

Sol.3 (A)

$$\frac{x^4 dx}{x+x^5} = \int \frac{(x^4+1)\,dx}{x+x^5} - \int \frac{dx}{x+x^5}$$

$$= \int \frac{(x^4+1)\,dx}{x(1+x^4)} - \int \frac{dx}{x(x^4+1)}$$

$$= \int \frac{dx}{x} - \int \frac{dx}{x+x^5}$$

= log x − f(x) − c_2 + c_1 = log x − f(x) + c

Where $c_1 - c_2$ = c = a new constant.

Sol.4 (A)

$$\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}}\, e^{\sin x} \cos x\,dx$$

$$= \int \frac{\sin x - \cos x}{\sin x - \cos x}\, e^{\sin x} \cos x\,dx$$

$$= \int e^{\sin x} \cos x\,dx = e^{\sin x} + c$$

Sol.5 (A)

Put x = 3 sin θ $\Rightarrow$ dx = 3 cos θ dθ, therefore

$$\int \frac{x^2}{(9-x^2)^{3/2}}\,dx = \int \frac{9\sin^2\theta}{(9-9\sin^2\theta)^{3/2}.3\cos\theta}\,d\theta$$

$$= \int \frac{27\sin^2\theta\cos\theta}{27\cos^3\theta}\,d\theta = \int \tan^2\theta\,d\theta$$

$$= \int (\sec^2\theta - 1)\,d\theta$$

= tan θ − θ + c

$$= \tan\left\{\sin^{-1}\left(\frac{x}{3}\right)\right\} - \sin^{-1}\left(\frac{x}{3}\right) + c$$

$$= \tan\tan^{-1}\left(\frac{\left(\frac{x}{3}\right)}{\sqrt{1-\left(\frac{x^2}{9}\right)}}\right) = \sin^{-1}\left(\frac{x}{3}\right) + c$$

$$= \frac{x}{\sqrt{9-x^2}} - \sin^{-1}\left(\frac{x}{3}\right) + c$$

Sol.6 (C)

$$\int \frac{dx}{(2\sin x + \cos x)^2} = \int \frac{dx}{\sin^2 x(2+\cot x)^2}$$

$$= \int \frac{\operatorname{cosec}^2 x\,dx}{(2+\cot x)^2}$$

Put (2 + cot x) = t $\Rightarrow$ − $\operatorname{cosec}^2 x$ dx = dt

$$= \int \frac{-dt}{t^2} = \frac{1}{t} + c = \frac{1}{2+\cot x} + c$$

Sol.7 (D)

$$\int \frac{1}{(\sin x + 4)(\sin x - 1)} dx$$

$$= \frac{1}{5}\int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)} dx$$

$$= \frac{1}{5}\int \frac{1}{\sin x - 1} dx - \frac{1}{5}\int \frac{1}{\sin x + 4} dx$$

$$= \frac{1}{5}\int \frac{2dt}{2t - 1 - t^2} - \frac{1}{5}\int \frac{2dt}{2t + 4(1+t^2)}$$

$$\left(\text{Putting } \tan\frac{x}{2} = t\right)$$

$$= -\frac{2}{5}\int \frac{dt}{t^2 - 2t + 1} - \frac{1}{10}\int \frac{dt}{t^2 + \frac{1}{2}t + 1}$$

$$= -\frac{2}{5}\int \frac{1}{(t-1)^2} dt - \frac{1}{10}\int \frac{dt}{\left(t+\frac{1}{4}\right)^2 + \left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{2}{5}\frac{1}{(t-1)} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right) + c$$

$$= \frac{2}{5}\cdot\frac{1}{\tan\frac{x}{2} - 1} - \frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right) + c$$

$$\therefore A = \frac{2}{5},\ B = \frac{-2}{5\sqrt{15}},\ f(x) = \frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$$

Sol.8 (B)

$$\int \frac{1}{x^2}\log(x^2 + a^2)\, dx$$

$$= \int x^{-2}\log(x^2 + a^2)\, dx$$

$$= \frac{-\log(x^2 + a^2)}{x} + \int \frac{2x}{(x^2 + a^2)x} dx + c$$

$$= \frac{-\log(x^2 + a^2)}{x} + \frac{2}{a}\tan^{-1}\frac{x}{a} + c$$