Dear student following is a Moderate level [O ● O] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-6(+3, –1) (All questions have only one option correct)

Q.1 $\int \frac{\log(x+1)-\log x}{x(x+1)}\,dx$ equals-

(A) $-\log\left(\frac{x+1}{x}\right)+c$

(B) $-\log\left[\log\left(\frac{x+1}{x}\right)\right]+c$

(C) $-\left(\frac{1}{2}\right)\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$

(D) $c-\frac{1}{2}[(\log(x+1)^2-(\log x)^2]$

Q.2 $\int e^{\sec x}\,.\,\sec^3 x\,.\,(\sin^2 x+\cos x+\sin x+\sin x\cos x)\,dx$ is equal to-

(A) $e^{\sin x}(\sec^2 x+\sec x\tan x)+c$

(B) $e^{\sec x}+c$

(C) $e^{\sec x}(\sec x+\tan x)+c$

(D) None of these

Q.3 If $\int \frac{\cos^4 x}{\sin^2 x}\,dx = A\cot x + B\sin 2x + C\frac{x}{2} + D$, then-

(A) $A=-2$

(B) $B=\frac{1}{4}$

(C) $C=3$

(D) $B=-\frac{1}{4}$, $C=-3$

Q.4 $\int \frac{(1+x)^2}{x+x^3}\,dx$ is equal to-

(A) $\log_e x+\log_e(1+x^2)+c$

(B) $\log_e x+\tan^{-1}x+c$

(C) $\log_e x+2\tan^{-1}x+c$

(D) None of these

Q.5 Let f(x) be a polynomial of degree 3 such that f(0) = 1, f(1) = 2 and 0 is a critical point of f(x) which does not have a local extremum at 0. Then $\int \frac{f(x)}{\sqrt{x^2+7}}\,dx$ equals-

(A) $\frac{1}{3}(x^2+7)^{3/2}-7(x^2+7)^{1/2}+\log\left|x+\sqrt{x^2+7}\right|+c$

(B) $\frac{1}{3}(x^2+7)^{3/2}+5(x^2+7)^{1/2}+\frac{2}{3}\log\left|x+\sqrt{x^2+7}\right|+c$

(C) $\frac{1}{3}(x^2+7)^{3/2}-\frac{3}{5}(x^2+7)^{1/2}+\log\left|x+\sqrt{x^2+7}\right|+c$

(D) None of these

Q.6 If $I=\int \frac{5+3x}{\sqrt{(x-1)(x+2)}}\,dx$, then I equals

(A) $3\sqrt{(x-1)(x+2)}+\frac{7}{2}\log\left|x+\frac{1}{2}\sqrt{(x-1)(x+2)}\right|+C$

(B) $3\sqrt{x^2-x-2}+7\log\left|x+\frac{1}{2}+\sqrt{x^2-x-2}\right|+C$

(C) $3\log\left|x+\frac{1}{2}\sqrt{(x-1)(x+2)}\right|+C$

(D) $3\sin^{-1}\sqrt{\frac{x-1}{x+2}}+C$

MATHEMATICS IIT JEE (SEPT. 2nd WEEK CLASS TEST 3) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	3	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭

ANSWER KEY

Que.	1	2	3	4	5	6
Ans.	C	C	D	C	A	A

SOLUTIONS

Sol.1 (C)

$$\int \frac{\log(x+1)-\log x}{x(x+1)}\,dx$$

$$= -\int (\log(x+1)-\log x)\,.\,\frac{1}{-x(x+1)}\,dx$$

$$= -\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + c$$

$$\left[\because \frac{d}{dx}[\log(x+1)-\log x] = \frac{1}{x+1}-\frac{1}{x} = -\frac{1}{(x+1)x}\right]$$

Sol.2 (C)

$$\int e^{\sec x}\,.\,\sec^3 x\,(\sin^2 x + \cos x + \sin x + \sin x\cos x)\,dx$$

$$= \int e^{\sec x}\,(\tan^2 x\,.\,\sec x + \sec^2 x + \tan x\,.\,\sec^2 x + \sec x\tan x)\,dx$$

$$= \int e^{\sec x}\,[\sec x\tan x(\sec x + \tan x) + \sec x\,(\sec x + \tan x)]\,dx$$

$$= \int e^{\sec x}\sec x\tan x\,(\sec x + \tan x)\,dx$$

II I

$$+ \int e^{\sec x}\,.\,\sec x\,(\sec x + \tan x)\,dx$$

$$= (\sec x + \tan x)\,.\,e^{\sec x} - \int (\sec x\tan x + \sec^2 x)\,e^{\sec x}\,dx$$

$$+ \int e^{\sec x}\,.\,(\sec^2 x + \sec x\tan x)\,dx + c$$

$$= e^{\sec x}\,(\sec x + \tan x) + c$$

Sol.3 (D)

$$\int \frac{\cos^4 x}{\sin^2 x}\,dx = \int \frac{(1-\sin^2 x)^2}{\sin^2 x}\,dx$$

$$= \int \frac{1+\sin^4 x - 2\sin^2 x}{\sin^2 x}\,dx$$

$$= \int \mathrm{cosec}^2 x\,dx + \int \sin^2 x\,dx - 2\int dx$$

$$= -\cot x + \int \frac{1-\cos 2x}{2}\,dx - 2x + \text{const.}$$

$$= -\cot x + \frac{x}{2} - \frac{\sin 2x}{4} - 2x + \text{const.}$$

$$= -\cot x - \frac{1}{4}\sin 2x - \frac{3}{2}x + D$$

$$= A\cot x + B\sin 2x + C\frac{3}{2} + D$$

$$\therefore A = -1,\ B = -\frac{1}{4},\ C = -3.$$

Sol.4 (C)

$$\int \frac{(1+x)^2}{x+x^3}\,dx = \int \frac{(1+x^2)+2x}{x+x^3}\,dx$$

$$= \int \frac{dx}{x} + 2\int \frac{dx}{1+x^2}$$

$$= \log_e x + 2\tan^{-1} x + c$$

Sol.5 (A)

Let $f(x) = ax^3 + bx^2 + cx + d$

$f(0) = 1 \Rightarrow d = 1$ ..... (1)

$f(1) = 2 \Rightarrow a + b + c + d = 2$

or $a + b + c = 1$ ..... (2)

Further $f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$x = 0$ is a critical point then

$f'(0) = 0 \Rightarrow c = 0$

But at x = 0 f(x) does not have a local extremum then $f''(0) = 0 \Rightarrow b = 0$

then eq. (2) becomes a = 1

Then $f(x) = x^3 + 1$

Now $\int \frac{f(x)}{\sqrt{x^2+7}}dx = \int \frac{x^3+1}{\sqrt{x^2+7}}\,dx$

$= \int \frac{x^2.x}{\sqrt{x^2+7}}dx + \int \frac{1}{\sqrt{x^2+\left(\sqrt{7}\right)^2}}\,dx$

$= \int \frac{(t^2-7)}{t}t\,dt + \int \frac{1}{\sqrt{x^2+7}}\,dx$

Putting $x^2 + 7 = t^2$ in Ist integral.

$= t^3/3 - 7t + \log\left|x+\sqrt{x^2+7}\right| + c$

$= \frac{1}{3}(x^2 + 7)^{3/2} - 7\sqrt{x^2+1}$

$+ \log\left|x+\sqrt{x^2+7}\right| + c$

Sol.6 (A)

Put $t = \frac{1}{2}(x - 1 + x + 2) = x + \frac{1}{2}$, so that $dx = dt$, $5 + 3x = 7/2 + 3t$, and

$(x - 1)(x + 2) = \left(t-\frac{1}{2}-1\right)\left(t-\frac{1}{2}+2\right)$

$= t^2 - \left(\frac{3}{2}\right)^2$

$\therefore \quad I = \int \frac{3t+7/2}{\sqrt{t^2-(3/2)^2}}\,dt = 3\sqrt{t^2-(3/2)^2}$

$+ \frac{7}{2}\log\left|t+\sqrt{t^2-(3/2)^2}\right| + C$

$= 3\sqrt{(x-1)(x+2)}$

$+ \frac{7}{2}\log\left|x+\frac{1}{2}+\sqrt{(x-1)(x+2)}\right| + C$