Dear student following is a Moderate level [O ● O] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3, –1) (Questions may have more than one option correct)

Q.1 $\int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx =$

(A) $e^{x/2}\cos\frac{x}{2}$ + C (B) $\sqrt{2}e^{x/2}\cos\frac{x}{2}$ + C

(C) $e^{x/2}\sin\frac{x}{2}$ + C (D) $\sqrt{2}e^{x/2}\sin\frac{x}{2}$ + C

Q.2 If $\int\left\{\frac{bx\cos(4x)-a\sin(4x)}{x^2}\right\}dx = \frac{a\sin(4x)}{x}$, then (a, b) =

(A) a = 1, b = 4 (B) a = –1, b = 4

(C) a = 1, b = $\frac{1}{4}$ (D) a = $\frac{1}{4}$, b = 1

Q.3 $\int e^{2x}\frac{1+\sin 2x}{1+\cos 2x}dx =$

(A) $e^{2x}\tan x$ + C (B) $e^{2x}\cot x$ + C

(C) $\frac{e^{2x}\tan x}{2}$ + C (D) $\frac{e^{2x}\cot x}{2}$ + C

Q.4 If $\int f(x)dx = g(x)$, then $\int f^{-1}(x)dx$ is equal to

(A) $g^{-1}(x)$ (B) $xf^{-1}(x) - g(f^{-1}(x))$

(C) $xf^{-1}(x) - g^{-1}(x)$ (D) $f^{-1}(x)$

Q.5 $\int\frac{\sec^{-1}x}{x^2\sqrt{x^2-1}}dx =$

(A) $\sec^{-1}x.\sin(\sec^{-1}x) - \cos(\sec^{-1}x) + c$

(B) $\sec^{-1}x.\sin(\cos^{-1}x) - \cos(\sin^{-1}x) + c$

(C) $\sec^{-1}x.\sin(\sec^{-1}x) + \cos(\sec^{-1}x) + c$

(D) $\sec^{-1}x.\sin(\cos^{-1}x) + \cos(\sec^{-1}x) + c$

Q.6 If $f(x) = \lim_{n\to\infty}\frac{x^n - x^{-n}}{x^n + x^{-n}}$, $0 < x < 1$, $n \in N$

then $\int(\sin^{-1}x)\,f(x)\,dx$ is equal to-

(A) $-\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$ + C

(B) $x\sin^{-1}x + \sqrt{1-x^2}$ + C

(C) Constant

(D) None of these

Q.7 If $f(x) = \lim_{n\to\infty} n^2(x^{1/n} - x^{1/(n+1)})$, $x > 0$ then

$\int x\,f(x)\,dx$ is equal to

(A) $x^2/2$ (B) 2

(C) $x^3/3$ (D) None of these

Q.8 If f(x) is polynomial of second degree such that f(0) = f(1) = 3f(2) = –3, then $\int\frac{f(x)}{x^3-1}dx$ is equal to

(A) $\log\frac{1+x+x^2}{x-1} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(B) $\log\frac{x-1}{1+x+x^2} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(C) $\log\frac{1+x+x^2}{x-1} + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(D) None of these

Q.9 $\int\frac{x^2+x-1}{x^2+x-6}dx =$

(A) $x + \log(x+3) + \log(x-2) + c$

(B) $x - \log(x+3) + \log(x-2) + c$

(C) $x - \log(x+3) - \log(x-2) + c$

(D) None of these

MATHEMATICS IIT JEE (SEPT. 3rd WEEK CLASS TEST 1) (INDEFINITE INTEGRATION) ANSWER KEY

Name : ..... Roll No. : .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	D	A,D	C	B	C	A	D	A	B

SOLUTIONS

Sol.1 (D)

$$\text{Let } I = \int e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

$$= \int e^{x/2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right).\frac{1}{\sqrt{2}}dx$$

$$\Rightarrow I\sqrt{2} = \int e^{x/2}\sin\left(\frac{x}{2}\right) + \int e^{x/2}\cos\left(\frac{x}{2}\right)dx,$$

Integrating second integral by parts,

$$I\sqrt{2} = \int e^{x/2}.\sin\left(\frac{x}{2}\right)dx + \left[e^{x/2}.\frac{\sin(x/2)}{1/2}\right.$$

$$\left. - \int\left(\frac{1}{2}e^{x/2}\right).2\sin\left(\frac{x}{2}\right)dx\right] + C_1$$

$$= 2e^{x/2}\sin\left(\frac{x}{2}\right) + C_1 \quad [\text{where } C = \frac{1}{\sqrt{2}}C_1]$$

$$\Rightarrow I = \sqrt{2}e^{x/2}\sin(x/2) + C$$

Sol.2 (A, D)

$$\text{Let } I=\int\left\{\frac{bx\cos(4x)-a\sin(4x)}{x^2}\right\}dx \ \ldots (1)$$

$$\text{and } I = \frac{a}{x}\sin(4x) \qquad \ldots (2)$$

$$(1) \Rightarrow \quad I= \int\frac{b\cos(4x)}{x}dx - \int\frac{a}{x^2}\sin(4x)dx,$$

Integrating by parts, the first integral only,

$$I = \frac{b\sin(4x)}{4x} + \int\frac{b\sin(4x)}{4x^2}dx$$

$$- \int\frac{a}{x^2}\sin(4x)dx$$

using (2), $\frac{a}{x}\sin(4x)$

$$= \frac{b\sin(4x)}{4x} + \left(\frac{b}{4}-a\right)\int\frac{\sin(4x)}{x^2}dx$$

Comparing, $a = \frac{b}{4}$, $\frac{b}{4} - a = 0 \Rightarrow a = \frac{b}{4}$

In each case. Only (A) and (D) satisfy the condition $a = \frac{b}{4}$

Sol.3 (C)

$$\text{Let } I = \int e^{2x}\frac{1+\sin2x}{1+\cos2x}dx$$

$$= \int e^{2x}\left[\frac{1}{1+\cos2x}+\frac{\sin2x}{1+\cos2x}\right]dx$$

$$= \int e^{2x}\left[\frac{\sec^2 x}{2}+\tan x\right]dx$$

$$= \frac{1}{2}\int e^{2x}\sec^2 x\ dx + \int e^{2x}\tan x\ dx$$

$$= \frac{e^{2x}\tan x}{2} - \int e^{2x}\tan x\ dx$$

$$+ \int e^{2x}\tan x\ dx$$

$$= \frac{e^{2x}\tan x}{2} + C$$

Sol.4 (B)

We have, $\int f(x)dx = g(x)$ ...(1)

$$\therefore \int f^{-1}(x).1\ dx$$

$$= f^{-1}(x)\int dx - \int\left\{\frac{d}{dx}f^{-1}(x)\int dx\right\}dx$$

$$= xf^{-1}(x) - \int x\frac{d}{dx}f^{-1}(x)dx$$

$$= xf^{-1}(x) - \int x\,d\left\{f^{-1}(x)\right\}$$

Let $f^{-1}(x) = t$

$\Rightarrow x = f(t)$ and $d\{f^{-1}(x)\} = dt$

$= xf^{-1}(x) - \int f(t)dt = xf^{-1}(x) - g(t)$

$= xf^{-1}(x) - g\{f^{-1}(x)\}$ [using (1)]

Sol.5 (C)

$$\text{Let } I = \int\frac{\sec^{-1}x}{x^2\sqrt{x^2-1}}dx$$

Put $\sec^{-1} x = t$

$$\Rightarrow \frac{1}{|x|\sqrt{x^2-1}}dx = dt$$

$$\therefore \quad I = \int \frac{t}{\sec t}dt$$

$$= \int t\cos t\, dt$$

$$= t\sin t - \int \sin t.1\, dt$$

$$= t\sin t + \cos t + c$$

$$= \sec^{-1}x\sin(\sec^{-1}x) + \cos(\sec^{-1}x) + c.$$

Sol.6 (A)

$$f(x) = \lim_{n\to\infty}\frac{x^{2n}-1}{x^{2n}+1} = -1 \ (0 < x < 1), \text{ so}$$

$$\int \sin^{-1}x\,(f(x))\,dx = -\int \sin^{-1}x\,dx$$

$$= -\left[x\sin^{-1}x + \sqrt{1-x^2}\right] + C$$

Sol.7 (D)

$$f(x) = \lim_{n\to\infty} n^2\, x^{1/(n+1)}\left[x^{\frac{1}{n}-\frac{1}{n+1}} - 1\right]$$

$$= \lim_{n\to\infty}\frac{x^{1/n+1}\left(x^{\frac{1}{n(n+1)}}-1\right)}{\frac{1}{n(n+1)}\times\frac{n(n+1)}{n^2}} = \log x$$

Hence $\int x\, f(x)\, dx = \int x\log x\, dx$

$$= \frac{x^2}{2}\log x - (1/4)\,x^2 + C$$

Sol.8 (A)

Given f(x) = polynomial of degree 2.. (1)

and f(0) = f(1) = 3f(2) = – 3 ... (2)

(1) $\Rightarrow$ f(x) = $ax^2 + bx + c$ ... (3)

Putting (2) in (3), we get

f(0) = c = –3, f(1) = a + b + c = –3 or a + b = 0

f(2) = 4a + 2b + c = –1

$\Rightarrow$ 4a + 2b – 3 = –1 $\Rightarrow$ 4a + 2b = 2

$\Rightarrow$ 4a – 2a = 2 $\Rightarrow$ a = 1, a + b = 0

$\Rightarrow$ b = –1

Now (3) $\Rightarrow$ $f(x) = x^2 - x - 3$

$$I = \int\frac{f(x)}{x^3-1}dx = \int\frac{(x^2-x-3)}{x^3-1}dx$$

$$= \int\frac{(x^2-x-3)}{(x-1)(x^2+x+1)}dx$$

By breaking it into partial fractions, we get

$$I = -\int\frac{dx}{x-1} + \int\frac{2(x+1)}{x^2+x+1}dx$$

$$= -\log(x-1) + \int\frac{(2x+1)}{x^2+x+1}dx + \int\frac{dx}{x^2+x+1}$$

$$= -\log(x-1) + \log(x^2+x+1) + \int\frac{dx}{x^2+x+1}$$

$$= -\log(x-1) + \log(x^2+x+1) + \int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

or, $I = \log\left(\frac{x^2+x+1}{x-1}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right) + c$

Sol.9 (B)

$$\int\frac{x^2+x-1}{x^2+x-6}\,dx = \int\left[1+\frac{5}{x^2+x-6}\right]dx$$

$$= \int\left[1+\frac{5}{(x+3)(x-2)}\right]dx$$

$$= \int dx + \int\frac{dx}{x-2} - \int\frac{dx}{x+3}$$

$$= x + \log(x-2) - \log(x+3) + c$$