SEPT. 3rd WEEK CLASS TEST 2 INDEFINITE INTEGRATION MATHEMATICS (FRESHER)

Dear student following is an Easy level [● ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-7(+3, –1) (All questions have only one option correct)

Q.1 If $\int \frac{1}{x\sqrt{5x^2-3}}dx = K\tan^{-1} f(x) + C$, then

(A) $f(x) = \sqrt{\frac{5}{3}x^2 - 1}$, $K = \frac{1}{\sqrt{5}}$

(B) $f(x) = \sqrt{\frac{5}{3}x^2 - 1}$, $K = \frac{1}{\sqrt{3}}$

(C) $f(x) = \frac{1}{2}\sqrt{5x^2-3}$, $K = \frac{1}{\sqrt{5}}$

(D) None

Q.2 $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + C$

(B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + C$

(C) $\frac{\sqrt{2x^4-2x^2+1}}{x} + C$

(D) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + C$

Q.3 $\int \cos^3 x\, e^{\log \sin x} dx$ is equal to

(A) $-\frac{\sin^4 x}{4} + C$ (B) $-\frac{\cos^4 x}{4} + C$

(C) $\frac{e^{\sin x}}{4} + C$ (D) None

Q.4 $\int \frac{dx}{x\sqrt{x^6-16}} =$

(A) $\frac{1}{3}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ (B) $\cosh^{-1}\left(\frac{x^3}{4}\right) + c$

(C) $\frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$ (D) $\sec^{-1}\left(\frac{x^3}{4}\right) + c$

Q.5 $\int \frac{(x^3+3x^2+3x+1)}{(x+1)^5}dx =$

(A) $-\frac{1}{(x+1)} + c$ (B) $\frac{1}{5}\log(x+1) + c$

(C) $\log(x+1) + c$ (D) $\tan^{-1} x + c$

Q.6 $\int \frac{\operatorname{cosec} x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx =$

(A) $\sin^2\left[1+\log\tan\frac{x}{2}\right] + c$

(B) $\tan\left[1+\log\tan\frac{x}{2}\right] + c$

(C) $\sec^2\left[1+\log\tan\frac{x}{2}\right] + c$

(D) $-\tan\left[1+\log\tan\frac{x}{2}\right] + c$

Q.7 If $\int \frac{\sqrt{x}}{x+1}dx = A\sqrt{x} + B\tan^{-1}\sqrt{x} + C$, then

(A) A = 1, B = 1 (B) A = 1, B = 2

(C) A = 1, B = 2 (D) A = 2, B = –2

MATHEMATICS IIT JEE (SEPT. 3rd WEEK CLASS TEST 2) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	◯	◯	◯	◯	4	◯	◯	◯	◯	7	◯	◯	◯	◯
2	◯	◯	◯	◯	5	◯	◯	◯	◯					
3	◯	◯	◯	◯	6	◯	◯	◯	◯					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	B	D	B	A	A	B	D

SOLUTIONS

Sol.1 (B)

On Differentiating both sides, we have

$$\frac{1}{x\sqrt{5x^2-3}} = K \frac{1}{(f(x))^2+1}\frac{d}{dx}f(x)$$

$$= K\frac{1}{\left(\sqrt{\frac{5}{3}x^2-1}\right)^2+1}\frac{d}{dx}\left(\sqrt{\frac{5}{3}x^2-1}\right)$$

$$= \frac{K}{\frac{5}{3}x^2-1+1}\cdot\frac{1}{2\sqrt{\frac{5}{3}x^2-1}}\cdot\frac{5}{3}(2x)$$

$$= \frac{K}{\frac{5}{3}x^2}\;\frac{\frac{5}{3}x}{\sqrt{\frac{5}{3}x^2-1}} = \frac{K}{\frac{x}{\sqrt{3}}.\sqrt{5x^2-3}}$$

$$= \frac{\sqrt{3}K}{x.\sqrt{5x^2-3}}$$

$$\Rightarrow f(x) = \sqrt{\frac{5}{3}x^2-1},\ K = \frac{1}{\sqrt{3}}$$

Sol.2 (D)

$$\text{Let } I = \int\frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$$

$$\text{Let } 2-\frac{2}{x^2}+\frac{1}{x^4} = z$$

$$\Rightarrow I = \frac{1}{4}\int\frac{dz}{\sqrt{z}} = \frac{1}{2}\times\sqrt{z}+c$$

$$\Rightarrow I = \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+C.$$

Sol.3 (B)

$$\int\cos^3x\,.\,\sin x\,dx$$

$$= -\int t^3\,dt \quad \text{let } \cos x = t \Rightarrow -\sin x\,dx = dt$$

$$= -\frac{t^4}{4}+C$$

$$= -\frac{\cos^4x}{4}+C$$

Sol.4 (A)

$$\int\frac{dx}{x\sqrt{x^6-16}}$$

Let $x^3 = t$, $3x^2\,dx = dt$, $dx = \frac{1}{3x^2}dt$

$$\therefore \int\frac{dx}{x\sqrt{x^6-16}} = \frac{1}{3}\int\frac{dt}{t\sqrt{t^2-4^2}}$$

$$= \frac{1}{3}\sec^{-1}\frac{t}{4}+C$$

$$= \frac{1}{3}\sec^{-1}\frac{x^3}{4}+C$$

Sol.5 (A)

$$\int\frac{(x^3+3x^2+3x+1)}{(x+1)^5}dx \Rightarrow \int\frac{(x+1)^3}{(x+1)^5}dx$$

$$\Rightarrow \int(x+1)^{-2}dx \Rightarrow \int\frac{1}{(x+1)^2}dx$$

$$\Rightarrow -\frac{1}{x+1}+C$$

Sol.6 (B)

$$I = \int \frac{\operatorname{cosec}x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx$$

Let $\left(1+\log\tan\frac{x}{2}\right) = t$

$\Rightarrow \frac{1}{\tan(x/2)} \cdot \sec^2 \frac{x}{2} \times \frac{1}{2}xdx = dt \Rightarrow$ cosecx dx = dt

$$\therefore\ I = \int \frac{1}{\cos^2 t}dt = \int \sec^2 t\,dt = \tan t + C$$

$$= \tan\left(1+\log\tan\frac{x}{2}\right) + C.$$

Sol.7 (D)

$$\int \frac{\sqrt{x}}{x+1}dx = \int \frac{x}{\sqrt{x}\left(\left(\sqrt{x}\right)^2+1\right)}\ dx$$

$$= 2\int \frac{t^2dt}{t^2+1} = 2\int \frac{t^2+1-1}{t^2+1}\ dt$$

let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$$= 2\int 1.dt\ -2\int \frac{1}{t^2+1}\ dt$$

$= 2t - 2\tan^{-1}t + c$

$= 2\sqrt{x} - 2\tan^{-1}\sqrt{x} + c$

$\Rightarrow A = 2,\ B = -2$