Dear student following is a Moderate level [O ● O] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3, –1) (Questions may have more than one option correct)

Q.1 If $\int$ cosec 2x dx = f(g(x)) + C, then

(A) Range g(x) = $(-\infty, \infty)$
(B) Dom f(x) = $(-\infty, \infty) \sim \{0\}$
(C) g'(x) = $\sec^2 x$
(D) f(x) = 1/x for all x $\in$ (0, ∞)

Q.2 If I = $\int \sec^2 x \operatorname{cosec}^4 x\, dx = K \cot^3 x + L \tan x + M \cot x + C$ then

(A) K = – 1/4 (B) L = 2
(C) M = –2 (D) None of these

Q.3 If I = $\int \dfrac{x^2+20}{(x\sin x+5\cos x)^2}dx$, then I equals

(A) $-\dfrac{x}{\cos x(x\sin x+5\cos x)}$ + tanx + C

(B) $\dfrac{x}{\sin x(x\sin x+5\cos x)}$ + cotx + C

(C) $(x\sin x - 5\cos x)^{-1}$ sinx + 7x + C
(D) None of these

Q.4 If I = $\int \tan^3 x \sec^5 x\, dx$, then I equals

(A) $\tan^7 x - \tan^5 x + C$

(B) $\dfrac{1}{7}\sec^7 x + \dfrac{1}{5}\sec^5 x + C$

(C) $\dfrac{1}{7}\sec^7 x - \dfrac{1}{5}\sec^5 x + C$

(D) None of these

Q.5 If I = $\int \dfrac{\sec^2 x - 7}{\sin^2 x}$ dx, then I equals

(A) $\dfrac{\tan x}{(\sin x)^7}$ + C (B) $\dfrac{\cos x}{\sin^7 x}$ + C

(C) $\dfrac{\sin x}{\cos^7 x}$ + C (D) None of these

Q.6 If I = $\int \dfrac{\sec^{-1}\sqrt{x} - \operatorname{cosec}^{-1}\sqrt{x}}{\sec^{-1}\sqrt{x} + \operatorname{cosec}^{-1}\sqrt{x}}dx$, then I equals

(A) $(4/\pi)\left(x\sec^{-1}\sqrt{x} - \sqrt{x-1}\right)$ + x + C

(B) $(4/\pi)\left(x\sec^{-1}\sqrt{x} + \sqrt{x-1}\right)$ – x + C

(C) $(4/\pi)\left(x\sec^{-1}\sqrt{x} - \sqrt{x-1}\right)$ – x + C

(D) None of these

Q.7 If I = $\int \dfrac{dx}{x\sqrt{x^4+1}}$, then I equals

(A) (1/4) log $\left|\dfrac{\sqrt{x^4+1}-1}{\sqrt{x^4+1}+1}\right|$ + C

(B) (1/4) log $\left|\dfrac{\sqrt{x^4+1}+1}{\sqrt{x^4+1}-1}\right|$ + C

(C) (1/4) $(x^4 + 1)^{3/2}$ + C
(D) (1/4) $(x^4 - 1)^{-3/2}$ + C

Q.8 If I = $\int \dfrac{\cos x\, dx}{\sqrt{a+b\cot^2 x}}$ (a > b > 0), then I equals

(A) $\dfrac{1}{a-b}\sqrt{a\sin^2 x + b\cos^2 x}$ + C

(B) $\dfrac{1}{a-b}\sqrt{a+b\cot^2 x}$ + C

(C) $\dfrac{1}{a-b}\left(\sqrt{a+b\cot^2 x}+x\right)$ + C

(D) $\dfrac{1}{a-b}\left(\sqrt{a+b\cot^2 x}-x\right)$ + C

MATHEMATICS IIT JEE (SEPT. 3rd WEEK CLASS TEST 4) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A,B,C	A,C	A	C	A	C	A	A

SOLUTIONS

Sol.1 (A, B, C)

$$\int \text{cosec } 2x dx = \frac{1}{2}\log|\text{cosec } 2x - \cot 2x| + C$$

$$= \frac{1}{2}\log\left|\frac{1-\cos 2x}{\sin 2x}\right| + C$$

$$= \frac{1}{2}\log|\tan x| + C$$

Thus $f(x) = \frac{1}{2}\log|x|$ and $g(x) = \tan x$.

Now, range $g(x) = (-\infty, \infty)$
and dom $f(x) = -(-\infty,\infty) - \{0\}$.
Moreover, $g'(x) = \sec^2 x$

and $f'(x) = \frac{1}{2x}$ for all $x \in (0, \infty)$.

Sol.2 (A, C)

Put $\tan x = t$,

$$I = \int (1 + t^2)\frac{(1+t^2)^2}{t^4}\frac{dt}{1+t^2}$$

$$= \int \frac{1+t^4+2t^2}{t^4}dt = \int\frac{dt}{t^4} + \int dt + \int\frac{2}{t^2}dt$$

$$= -\frac{1}{4}(\tan x)^{-3} + \tan x - 2(\tan x)^{-1} + C$$

$$= -\frac{1}{4}\cot^3 x + \tan x - 2\cot x + C$$

Therefore, $K = -1/4$, $L = 1$, $M = -2$.

Sol.3 (A)

We can write

$$I = \int\frac{x^8(x^2+20)}{(x^5\sin x + 5x^4\cos x)^2}dx$$

$$= \int\frac{(x^5+20x^3)\cos x}{(x^5\sin x+5x^4\cos x)^2}\left(\frac{x^5}{\cos x}\right)dx$$

Note that

$$\frac{d}{dx}[x^5\sin x + 5x^4\cos x] = (x^5 + 20x^3)\cos x$$

Integrating by parts, we get

$$I = -\frac{x^5}{\cos x(x^5\sin x + 5x^4\cos x)}$$

$$+ \int\frac{5x^4\cos x + x^5\sin x}{(x^5\sin x+5x^4\cos x)}\frac{dx}{\sec^2 x}$$

$$= -\frac{x}{\cos x(x\sin x + 5\cos x)} + \tan x + C$$

Sol.4 (C)

Put $\sec x = t$, so that

$$I = \int (t^2-1)t^4 dt = \frac{1}{7}t^7 - \frac{1}{5}t^5 + C$$

Sol.5 (A)

$$I = \int\frac{\sec^2 x}{\sin^7 x}dx - 7\int\frac{dx}{\sin^7 x}$$

$$= \frac{\tan x}{\sin^7 x} + 7\int\frac{\tan x\cos x}{\sin^8 x}dx - 7\int\frac{dx}{\sin^7 x}$$

$$= \frac{\tan x}{\sin^7 x} + C$$

Sol.6 (C)

Using $\sec^{-1}\sqrt{x} + \text{cosec}^{-1}\sqrt{x} = \pi/2$, we get

$$I = \int\frac{2\sec^{-1}\sqrt{x} - \pi/2}{\pi/2}dx = \frac{4}{\pi}I_1 - x$$

Where $I_1 = \int \sec^{-1}\sqrt{x}\,dx$

Put $\sqrt{x} = \sec\theta$

$$I_1 = \int \theta(2\sec^2\theta \tan\theta)\, d\theta$$

$$= \theta\sec^2\theta - \int \sec^2\theta\, d\theta$$

$$= \theta\sec^2\theta - \tan\theta$$

$$= x\sec^{-1}\sqrt{x} - \sqrt{x-1}$$

$$\therefore\ I = \frac{4}{\pi}\left(x\sec^{-1}\sqrt{x} - \sqrt{x-1}\right) - x + C$$

Sol.7 (A)

Put $x^4 + 1 = t^2$, so that

$$I = \frac{1}{2}\int \frac{t\,dt}{(t^2-1)t} = \frac{1}{4}\log\left|\frac{t-1}{t+1}\right| + C$$

Sol.8 (A)

$$I = \int \frac{\sin x \cos x}{\sqrt{a\sin^2 x + b\cos^2 x}}\, dx$$

Put $a\sin^2 x + b\cos^2 x = t^2$

$\Rightarrow (a - b)\sin x \cos x\, dx = t\, dt$

$$I = \frac{1}{a-b}\int dt = \frac{1}{a-b}\, t + C$$