Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9 (+3, –1). (All questions have only one option correct)

Q.1 Let $f(x) = \int e^x (x-1)(x-2)\,dx$, Then f decrease in the interval
(A) $(-\infty, -2)$ (B) $(-2, -1)$
(C) $(1, 2)$ (D) $(2, +\infty)$

Q.2 $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ is equal to

(A) $\left(1+\frac{1}{x^4}\right)^{1/4} + C$ (B) $(1+x^4)^{1/4} + C$

(C) $\left(1-\frac{1}{x^4}\right)^{1/4} + C$ (D) $-\left(1+\frac{1}{x^4}\right)^{1/4} + C$

Q.3 If $\int \frac{dx}{x^{22}(x^7-6)} = A\{\ln(P)^6 + 9p^2 - 2p^3 - 18\}$ + c then

(A) $A = \frac{1}{9072}$, $p = \left(\frac{x^7-6}{x^7}\right)$

(B) $A = \frac{1}{54432}$, $p = \left(\frac{x^7-6}{x^7}\right)$

(C) $A = \frac{1}{54432}$, $p \left(\frac{x^7}{x^7-6}\right)$

(D) $A = \frac{1}{9072}$, $p = \left(\frac{x^7-6}{x^7}\right)^{-1}$

Q.4 If $\int \frac{dx}{\sin^4 + \cos^4 x} = \frac{1}{\sqrt{2}}\tan^{-1} f(x) + C$ then
(A) $f(x) = \tan x - \cot x$
(B) $f(\pi/4) = 0$
(C) $f(x)$ is continuous on R
(D) $f(x) = \frac{1}{2}(\tan x - \cot x)$

Q.5 If $f(x) = \lim_{n\to\infty} n^2 (x^{1/n} - x^{1/(n+1)})$, $x > 0$ then $\int x f(x)\,dx$ is equal to
(A) $x^2/2$ (B) 0
(C) $x^2 \log x - \frac{1}{2}x^2 + C$
(D) None of these

Q.6 The function f whose graph passes through (0, 7/3) and whose derivative is $x\sqrt{1-x^2}$ is given by
(A) $f(x) = (1/3)((1-x^2)^{3/2} + 7)$
(B) $f(x) = (3/2)[\sin^{-1}x + 6]$
(C) $f(x) = -(1/3)[(1-x^2)^{3/2} - 8]$
(D) $f(x) = -(2/3)[(1-x^2)^{3/2} - 8]$

Q.7 If $\int \frac{\cos 4x + 1}{\cot x - \tan x} = K\cos 4x + C$, then
(A) $K = -1/2$ (B) $K = -1/8$
(C) $K = -1/5$ (D) None of these

Q.8 If the antiderivative of $f(x) = \frac{\sin x}{\sin^2 x + 4\cos^2 x}$ is $(1/\sqrt{3})\tan^{-1}$ $(1/\sqrt{3}\ g(x)) + C$ then g(x) is equal to
(A) secx (B) tanx
(C) sinx (D) cosx

Q.9 The vaue of the integral $\int \frac{dx}{x^n(1+x^n)^{1/n}}$, $n \in N$ is

(A) $\frac{1}{(1-n)}\left(1+\frac{1}{x^n}\right)^{1-1/n} + c$

(B) $\frac{1}{(1+n)}\left(1-\frac{1}{x^n}\right)^{1+1/n} + c$

(C) $-\frac{1}{(1-n)}\left(1-\frac{1}{x^n}\right)^{1-1/n} + c$

(D) $-\frac{1}{(1+n)}\left(1+\frac{1}{x^n}\right)^{1+1/n} + c$

MATHEMATICS IIT JEE (SEPT. 4th WEEK CLASS TEST 1) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	C	D	B	B	D	C	B	A	A

SOLUTIONS

Sol.1 (C)

$f(x) = \int e^x (x-1)(x-2)\, dx$

For decreasing function, $f'(x) < 0$

$\Rightarrow e^x (x-1)(x-2) < 0$

$\Rightarrow (x-1)(x-2) < 0$

$\Rightarrow 1 < x < 2$

$\therefore e^x > 0 \ \forall \ x \in R$

Sol.2 (D)

$$\int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

Put $1 + \frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}\, dx = dt$

So, integeral is

$$I = -\frac{1}{4}\int \frac{dt}{t^{3/4}} = -t^{1/4} + c = -\left(1+\frac{1}{x^4}\right)^{1/4} + c$$

Sol.3 (B)

Let $I = \int \frac{dx}{x^{29}\left(1-\frac{6}{x^7}\right)}$

Put $1 - \frac{6}{x^7} = p \Rightarrow \frac{42}{x^8} dx = dp$

and $x^7 = \frac{6}{1-p}$

$$\therefore I = \frac{1}{42}\int \frac{(1-p)^3}{(6)^3 p}\, dp$$

$$= \frac{1}{(42)(216)}\int \frac{1-p^3-3p+3p^2}{p}\, dp$$

$$= \frac{1}{9072}\int \left(\frac{1}{p} - p^2 - 3 + 3p\right) dp$$

$$= \frac{1}{9072}\left(\log p - \frac{p^3}{3} - 3p + \frac{3}{2}p^2\right) + c$$

$$= \frac{1}{54432}(6\ell np - 2p^3 - 18p + 9p^2) + c$$

$$= \frac{1}{54432}(\ell np^6 + 9p^2 - 2p^3 - 18p) + c$$

$A = \frac{1}{54432}$, $p = \left(\frac{x^7-6}{x^7}\right)$

Sol.4 (B)

Dividing the numerator and denominator by $\cos^4 x$, the given integral can be written as

$$\int \frac{\sec^2 x(1+\tan^2 x)}{\tan^4 x + 1} dx = \int \frac{1+t^2}{1+t^4} dt$$

$(t = \tan x)$

$$= \int \frac{1+1/t^2}{t^2+1/t^2} dt = \int \frac{1+1/t^2}{\left(t-\frac{1}{t}\right)^2+2} dt$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}\left(t-\frac{1}{t}\right) + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{\tan x - \cot x}{\sqrt{2}} + C$$

Thus $f(x) = \frac{\tan x - \cot x}{\sqrt{2}}$ so $f\left(\frac{\pi}{4}\right) = 0$

Sol.5 (D)

$$f(x) = \lim_{n\to\infty} n^2\, x^{1/(n+1)} \left[x^{\frac{1}{n}-\frac{1}{n+1}} - 1\right]$$

$$= \lim_{n\to\infty} \frac{x^{1/(n+1)}\left(x^{\frac{1}{n(n+1)}} - 1\right)}{\frac{1}{n(n+1)} \times \frac{n(n+1)}{n^2}} = \log x$$

Hence $\int x\, f(x)\, dx = \int x \log x\, dx$

$$= \frac{x^2}{2}\log x - \frac{1}{4}x^2 + C$$

Sol.6 (C)

$$f(x) = \int x\sqrt{1-x^2}\, dx$$

$$= -\frac{1}{2}\int (-2x)\sqrt{1-x^2}\, dx$$

$$= -\frac{1}{3}(1-x^2)^{3/2} + C$$

So $7/3 = -1/3 + C \Rightarrow C = 8/3$

Therefore, $f(x) = -(1/3)[(1-x^2)^{3/2} - 8]$

Sol.7 (B)

$$\int \frac{\cos 4x + 1}{\cot x - \tan x}\, dx$$

$$= \int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x} .\ \sin x \cos x\, dx$$

$$= \int \cos 2x \sin 2x\, dx$$

$$= \frac{1}{2}\int \sin 4x\, dx = -\frac{1}{8}\cos 4x + C$$

Hence $K = -1/8$

Sol.8 (A)

$$f(x) = \frac{\sin x}{\sin^2 x + 4\cos^2 x}$$

$$= \frac{\tan x \sec x}{\tan^2 x + 4} = \frac{\tan x \sec x}{\sec^2 x + 3}$$

Putting $\sec x = t$, $\sec x \tan x\, dx = dt$ so

$$\int f(x)\, dx = \int \frac{dt}{t^2+3}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sec x}{\sqrt{3}} + C$$

Sol.9 (A)

Let $I = \int \frac{dx}{x^n(1+x^n)^{1/n}} = \int \frac{dx}{x^{n+1}\left(1+\frac{1}{x^n}\right)^{1/n}}$

Put $1 + \frac{1}{x^n} = t \Rightarrow -\frac{n}{x^{n+1}}dx = dt$

$$\Rightarrow \frac{1}{x^{n+1}}dx = -\frac{1}{n}dt$$

$$\therefore I = \frac{1}{n}\int \frac{dt}{t^{1/n}} = -\frac{1}{n}\int t^{-1/n}\, dt$$

$$= -\frac{1}{n}\left[\frac{t^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right] + c$$

$$= \frac{1}{1-n}\left(1+\frac{1}{x^n}\right)^{1-1/n} + c$$