Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-7 (+3, –1). (Questions may have more than one option correct)

Q.1 $\int \left(\frac{\cos x - \cos^3 x}{1-\cos^3 x}\right)^{1/2} dx$ is equal to

(A) $\frac{1}{3}\sin^{-1}(\cos^{3/2}x) + c$

(B) $-\frac{2}{3}\sin^{-1}(\cos^{3/2}x) + c$

(C) $\frac{4}{7}\sin^{-1}(\cos^{3/2}x) + c$

(D) None of these

Q.2 $\int \frac{\sin^8 - \cos^8 x}{1-2\sin^2 x\cos^2 x} dx$ is equal to

(A) $\sin 2x + c$ (B) $-\frac{1}{2}\sin 2x + c$

(C) $\frac{1}{2}\sin 2x + c$ (D) $-\sin 2x + c$

Q.3 The value of $\int e^x \frac{1+nx^{n-1}-x^{2x}}{(1-x^n)\sqrt{1-x^{2n}}} dx$ is

(A) $\frac{e^x\sqrt{1-x^n}}{1-x^n} + c$ (B) $e^x \frac{\sqrt{1+x^{2n}}}{1-x^{2n}} + c$

(C) $\frac{e^x\sqrt{1-x^{2n}}}{1-x^{2n}} + c$ (D) $e^x \frac{\sqrt{1-x^{2n}}}{1-x^n} + c$

Q.4 $\int \frac{dx}{\sqrt[4]{1+x^4}} = \frac{1}{2}\left(\frac{1}{2}\log\frac{1+z}{1-z} - \tan^{-1} z\right) + c$ where

(A) $z = \frac{\sqrt[4]{1+x^4}}{x}$ (B) $z = \frac{x}{\sqrt[4]{1+x^4}}$

(C) $z = -\frac{\sqrt[4]{1+x^4}}{x}$ (D) None of these

Q.5 $\int \frac{dx}{\sin^6 x + \cos^6 x}$ is equal to

(A) $\tan^{-1}(\tan x + \cot x) + c$

(B) $\tan^{-1}(\cot x - \tan x) + c$

(C) $\tan^{-1}(\tan x - \cot x) + c$

(D) None of these

Passage : Using the following data, solve **Q.6** & **7**

Consider the integral $\int \frac{\phi(x)dx}{\sqrt{(ax^2+bx+c)}}$ where $\phi(x)$ is a polynomial in x. If $\phi(x)$ is polynomial of degree n, then there exists a polynomial f(x) of degree (n–1) and a constant D such that

$$\int \frac{\phi(x)dx}{\sqrt{(ax^2+bx+c)}} dx = f(x)\sqrt{ax^2+bx+c} + D\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

Q.6 Differentiating both sides with respect to x and multiply by $\sqrt{ax^2+bx+c}$ we get

(A) $\phi(x) = f'(x)(ax^2+bx+c) + \frac{1}{2}(2ax+b)f(x) + D$

(B) $\phi(x) = f'(x)(ax^2+bx+c) - \frac{1}{2}(2ax+b)f(x) + D$

(C) $\phi(x) = \frac{1}{2}f'(x)(ax^2+bx+c) + \frac{1}{2}(2ax+b)f(x) + D$

(D) None of these

Q.7 Now apply this method to evaluate the given integral. If $\int \frac{(x^3+4x^2-6x+3)dx}{\sqrt{5+6x-x^2}}$

$= (Ax^2+Bx+C)\sqrt{(5+6x-x^2)} + D\sin^{-1}\left(\frac{3-x}{\sqrt{14}}\right)$, then

(A) A = 1/3 (B) B = 9/2

(C) C = 227/6 (D) D = –139

MATHEMATICS IIT JEE (SEPT. 4th WEEK CLASS TEST 2) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○					
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	B	B	D	A	C	A	All

SOLUTIONS

Sol.1 (B)

$$\int\left(\frac{\cos x-\cos^3 x}{1-\cos^3 x}\right)^{1/2} dx$$

$$= \int\sqrt{\frac{\cos x(1-\cos^2 x)}{1-\cos^3 x}}\, dx \;=\; \int\frac{\sqrt{\cos x}\,\sin dx}{\sqrt{1-\cos^3 x}}$$

Put $\cos^{3/2} x = z$

$\therefore\ \frac{3}{2}\cos^{1/2}x.(-\sin x)\ dx = dz$

$\therefore$ given integral $= -\frac{2}{3}\int\frac{dz}{\sqrt{1-z^2}}$

$= -\frac{2}{3}\sin^{-1} z + c$

$= -\frac{2}{3}\sin^{-1}(\cos^{3/2}x) + c$

Sol.2 (B)

$$\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$$

$$= \int\frac{(\sin^4 x-\cos^4 x)(\sin^4 x+\cos^4 x)}{1-2\sin^2 x\cos^2 x}dx$$

$$= \int\frac{(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)\,[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x]}{1-2\sin^2 x\cos^2 x}dx$$

$$= \int\frac{(\sin^2 x-\cos^2 x)(1)(1-2\sin^2 x\cos^2 x)}{1-2\sin^2 x\cos^2 x}dx$$

$$= -\int \cos 2x\ dx = -\frac{\sin 2x}{2} + c$$

Sol.3 (D)

$$\int e^x\,\frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}}dx$$

$$= \int e^x\,\frac{(1-x^{2n})+nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}dx$$

$$= \int e^x\left[\sqrt{\frac{1+x^n}{1-x^n}} + n.x^{n-1}\frac{1}{(1-x^n)^2}\sqrt{\frac{1-x^n}{1+x^n}}\right]dx$$

$$= e^x\sqrt{\frac{1+x^n}{1-x^n}} + c \quad \left|\int e^x\,(f(x)+f'(x))\,dx \text{ Type}\right.$$

$$= e^x\,\frac{\sqrt{1-x^{2n}}}{1-x^n} + c$$

Sol.4 (A)

Put $1 + x^4 = x^4 z^4 \Rightarrow x^4 = \frac{1}{z^4-1}$

$\Rightarrow dx = \frac{-z^3dz}{x^3(z^4-1)^2}$

$$\therefore \int\frac{dx}{\sqrt[4]{1+x^4}} = \int\frac{1}{zx}.\ \frac{-z^3}{x^3(z^4-1)^2}\ dz$$

$$= -\int\frac{z^2}{x^4(z^4-1)^2}dz = -\int\frac{z^2dz}{z^4-1}$$

$$= \int\frac{z^2}{(1+z^2)(1-z^2)}dz$$

$$= \frac{1}{2}\int\left(\frac{1}{1-z^2}-\frac{1}{(1+z^2)}\right)dz$$

$$= \frac{1}{2}\left(\frac{1}{2}\log\frac{1+z}{1-z}-\tan^{-1} z\right) + c$$

Where $z = \frac{1}{x}.\ \sqrt[4]{1+x^4}$

Sol.5 (C)

$$\int \frac{dx}{\sin^6 x + \cos^6 x} = \int \frac{\sec^6 x}{1+\tan^6 x} dx$$

$$= \int \frac{(1+\tan^2 x)^2 \sec^2 x}{1+\tan^6 x} dx \quad \text{Put } \tan x = t$$

$$\therefore \sec^2 x \, dx = dt$$

$$= \int \frac{(1+t^2)^2 dt}{1+t^6} = \int \frac{(1+t^2)^2 dt}{(1+t^2)(1-t^2+t^4)} dt$$

$$= \int \frac{1+t^2}{1-t^2+t^4} dt = \int \frac{1+\frac{1}{t^2}}{t^2-1+\frac{1}{t^2}} dt$$

Put $t - \frac{1}{t} = z \quad \therefore \left(1+\frac{1}{t^2}\right)dt = dz$

$t^2 + \frac{1}{t^2} = z^2 + 2$

$$= \int \frac{dz}{z^2+1} = \tan^{-1} z + c$$

$$= \tan^{-1}\left(t - \frac{1}{t}\right) + c$$

$$= \tan^{-1}(\tan x - \cot x) + c$$

Sol.6 (A)

Sol.7 (All)