Dear student following is an Easy level [● O O] test paper. Score of 18 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8(+3, –1) *(All questions have only one option correct)*

Q.1 If $f(x) = \dfrac{1}{\cos^2 x\sqrt{1+\tan x}}$ then its antiderivative F(x) (given that F(0) = 4) is-

(A) $\sqrt{1+\tan x} + 4$ (B) $\frac{2}{3}(1 + \tan x)^{3/2}$

(C) $2\left(\sqrt{1+\tan x}+1\right)$ (D) None of these

Q.2 Let $f(x) = \dfrac{1}{4-3\cos^2 x+5\sin^2 x}$ and if its antiderivative $F(x) = (1/3)\tan^{-1}(g(x)) + C$ then g(x) is equal to-

(A) 3 tan x (B) $(\sqrt{2})$ tan x

(C) 2 tan x (D) None of these

Q.3 The value of $\int \dfrac{dx}{\sqrt{x}+\sqrt[3]{x}}$ is-

(A) $3\sqrt{x} + 3(\sqrt[3]{x}) - 6(\sqrt[6]{x}) + 6\log(\sqrt[6]{x}+1) + C$

(B) $2\sqrt{x} + 6\sqrt[6]{x} - 6\log(\sqrt[6]{x}+1) + C$

(C) $2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x}+1) + C$

(D) None of these

Q.4 $\int \dfrac{dx}{\sqrt{2-3x-x^2}} = fog(x) + C$ then-

(A) $f(x) = \sin^{-1} x$, $g(x) = \dfrac{2x-3}{\sqrt{17}}$

(B) $f(x) = \tan^{-1} x$, $g(x) = \dfrac{2x+3}{\sqrt{17}}$

(C) $f(x) = \sin^{-1} x$, $g(x) = \dfrac{2x+3}{\sqrt{17}}$

(D) None of these

Q.5 $\int \dfrac{dx}{\sin x+\cos x}$ is equal to-

(A) log tan $(\pi/4 + x/8) + C$

(B) $\sqrt{2}$ log $|\tan(x/2 + \pi/8)| + C$

(C) $\dfrac{1}{\sqrt{2}}$ log $\left|\tan\left(\dfrac{x}{2}+\dfrac{\pi}{8}\right)\right| + C$

(D) $\sqrt{2}$ log $\left|\tan\left(\dfrac{x}{4}-\dfrac{\pi}{8}\right)\right| + C$

Q.6 $\int \dfrac{\cos x}{\cos x+\sin x}\,dx$ equals-

(A) $\dfrac{x^2}{2}$ + log (sin x + cos x) + c

(B) $\dfrac{1}{2}$ [x + log (sin x + cos x)] + c

(C) x + 2 log (sin x + cos x) + c

(D) None of these

Q.7 $\int \dfrac{d(x^2+1)}{\sqrt{x^2+2}}$ equals-

(A) $\sqrt{x^2+2} + k$ (B) $2\sqrt{x^2+2} + k$

(C) $\dfrac{1}{(x^2+2)^{3/2}} + k$ (D) None of these

Q.8 The value of integral $\int \dfrac{x}{1+x\tan x}\,dx$ is equal to-

(A) log |cos x + x|

(B) log |cos x + x sin x| + c

(C) log |x cos x + sin x| + c

(D) None of these

MATHEMATICS IIT JEE (AUGUST 5th WEEK CLASS TEST 1) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	C	A	C	C	C	B	B	B

SOLUTIONS

Sol.1 (C)

$$F(x) = \int \frac{dx}{\cos^2 x\sqrt{1+\tan x}} + C$$

$$= \int \frac{1}{\sqrt{1+t}} dt + C \quad (t = \tan x)$$

$$= 2\left(\sqrt{1+t}\right) + C = 2\sqrt{1+\tan x} + C$$

Since $4 = F(0) = 2 + C \Rightarrow C = 2$. Hence

$$F(x) = 2\left(\sqrt{1+\tan x}+1\right)$$

Sol.2 (A)

$$F(x) = \int \frac{dx}{4-3\cos^2 x+5\sin^2 x} + C$$

$$= \int \frac{\sec^2 x dx}{4(1+\tan^2 x)-3+5\tan^2 x} + C$$

$$= \int \frac{dt}{9t^2+1} + C \qquad (t = \tan x)$$

$$= \frac{1}{3}\tan^{-1} 3t + C$$

Therefore $g(x) = 3 \tan x$

Sol.3 (C)

We want a substitution that will allow us to find both the square root and the cube root without getting fractional exponents. Thus we want a substitution of the form $x = u^k$ where k is a multiple of 2 and 3. Let us use the least common multiple 6.

Let $x = u^6$, $dx = 6u^5\, du$

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{6u^5du}{u^3+u^2} = 6\int \frac{u^3}{u+1}du$$

$$= 6\int\left(u^2-u+1-\frac{1}{u+1}\right)du$$

$$= 2u^3 - 3u^2 + 6u - 6\log(u+1)+C$$

$$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x}+1)+C$$

Sol.4 (C)

$$\int \frac{dx}{\sqrt{2-3x-x^2}} = \int \frac{dx}{\sqrt{-(x^2+3x-2)}}$$

$$= \int \frac{dx}{\sqrt{-\left((x+3/2)^2-17/4\right)}}$$

$$= \int \frac{dx}{\sqrt{17/4-(x+3/2)^2}}$$

$$= \sin^{-1}\frac{x+3/2}{\sqrt{17}/2} + C = \sin^{-1}\frac{2x+3}{\sqrt{17}} + C$$

Therefore, $g(x) = \frac{2x+3}{\sqrt{17}}$ and $f(x) = \sin^{-1}x$.

Sol.5 (C)

$$\int \frac{dx}{\sin x+\cos x}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sin(\pi/4)\sin x+\cos(\pi/4)\cos x}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\cos(x-\pi/4)}$$

$$= \frac{1}{\sqrt{2}}\int \sec(x-\pi/4)\, dx$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{\pi}{8}\right)\right| + C$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)\right| + C$$

Sol.6 (B)

We have $\frac{\cos x}{\cos x + \sin x}$

$= \frac{1}{2} \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\cos x + \sin x}$

$\Rightarrow \frac{\cos x}{\cos x + \sin x} = \frac{1}{2}.1 + \frac{1}{2}.\frac{\cos x - \sin x}{\sin x + \cos x}$

$\Rightarrow \int \frac{\cos x}{\cos x + \sin x} dx$

$= \frac{x}{2} + \frac{1}{2} \log (\sin x + \cos x) + C$

Sol.7 (B)

$\int \frac{d(x^2+1)}{\sqrt{x^2+2}} = \int \frac{d(x^2+2)}{\sqrt{x^2+2}}$ as

$d(x^2 + 2) = d(x^2 + 1)$

$= \int \frac{dt}{\sqrt{t}} = 2\sqrt{x^2+2} + k$

Sol.8 (B)

$\int \frac{x}{1 + x\tan x} dx = \int \frac{x\cos x}{\cos x + x\sin x} dx$

$= \int \frac{1}{t} dt$ where $t = \cos x + x \sin x$

$= \log |t| + c = \log |\cos x + x \sin x| + c$