Dear student following is an Easy level [● O O] test paper. Score of 21 Marks in 10 Minutes would be a satisfactory performance. Questions 1-9(+3, –1) (All questions have only one option correct)

Q.1 $\int \frac{\cos 13x - \cos 14x}{1 + 2\cos 9x} dx =$

(A) $\frac{\sin 4x}{4} + \frac{\sin 5x}{5} + k$

(B) $\frac{\cos 4x}{4} - \frac{\cos 5x}{5} + k$

(C) $\frac{\sin 4x}{4} - \frac{\sin 5x}{5} + k$

(D) $\frac{\cos 4x}{4} + \frac{\cos 5x}{5} + k$

Q.2 $\int \frac{7x^8 + 8x^7}{(1 + x + x^8)^2} dx =$

(A) $\frac{x^7}{1 + x + x^8} + k$ (B) $\frac{x}{1 + x + x^8} + k$

(C) $\frac{x^8}{1 + x + x^8} + k$ (D) None

Q.3 $\int \frac{1 + \log x}{3 + x\log x} dx =$

(A) $\log |3 + x \log x| + c$

(B) $\log \left|\frac{3}{2} + \frac{x}{2}\log x\right| + c$

(C) Above two

(D) $\log |3x \log x + 1| + c$

Q.4 $\int e^x \frac{(x^3 - 3x^2 + 5x - 1)}{(x^2 + 1)^2}$ dx is (apart from const)

(A) $e^x \left(\frac{x - 2}{x^2 + 1}\right)$ (B) $e^x \frac{x + 2}{x^2 + 1}$

(C) $e^x \frac{2x + 1}{x^2 + 1}$ (D) $e^x \frac{2x - 1}{x^2 + 1}$

Q.5 $\int \left(\frac{\sec^2 x - 7}{\sin^7 x}\right) dx =$

(A) $\frac{1}{(\sin x)^7} + c$ (B) $\frac{\tan x}{(\sin x)^7} + c$

(C) $\frac{\sec x \tan x}{(\sin x)^7} + c$ (D) None of these

Q.6 $\int [\sin(\log x) + \cos(\log x)] dx =$

(A) x sin (log x) + c

(B) x cos (log x) + c

(C) sin (log x) / x + c

(D) cos (log x) / x + c

Q.7 $\int x^x (1 + \log x) dx =$

(A) $x^x \log x + k$ (B) $e^{x^x} + k$

(C) $x^x + k$ (D) None of these

Q.8 $\int \frac{dx}{x^2 + 2x + 2}$ equals-

(A) $\sin^{-1} (x + 1) + c$

(B) $\sin h^{-1} (x + 1) + c$

(C) $\tan h^{-1} (x + 1) + c$

(D) $\tan^{-1} (x + 1) + c$

Q.9 $\int \sqrt{x}\, e^{\sqrt{x}}$ dx equals-

(A) $2\sqrt{x} - e^{\sqrt{x}} - 4\sqrt{x}\, e^{\sqrt{x}} + c$

(B) $(2x - 4\sqrt{x} + 4)\, e^{\sqrt{x}} + c$

(C) $(2x - 4\sqrt{x} + 4)\, e^{\sqrt{x}} + c$

(D) $(1 - 4\sqrt{x})\, e^{\sqrt{x}} + c$

MATHEMATICS IIT JEE (AUGUST 5th WEEK CLASS TEST 2) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	C	C	C	A	B	A	C	D	C

SOLUTIONS

Sol.1 (C)

Transform the integral as

$$\frac{\cos 13x - \cos 14x}{1 + 2\cos 9x}$$

$$= \frac{\cos 13x - \cos 14x}{1 + 2\cos 9x} \times \frac{\sin\frac{9}{2}x}{\sin\frac{9}{2}x}$$

$$= \frac{2\sin\frac{27x}{2}\sin\frac{x}{2}.\sin\frac{9}{2}x}{\sin\frac{9}{2}x + \sin\frac{27x}{2} - \sin\frac{9x}{2}}$$

$$= 2\sin\frac{x}{2}\sin\frac{9}{2}x = \cos 4x - \cos 5x$$

Now $\int \frac{\cos 13x - \cos 14x}{1 + 2\cos 9x}dx$

$$= \int \cos 4x\,dx - \int \cos 5x\,dx$$

$$= \frac{\sin 4x}{4} - \frac{\sin 5x}{5} + k$$

Sol.2 (C)

Let $I = \int \frac{7x^8 + 8x^7}{(1 + x + x^8)^2}dx$

$$= \int \frac{7x^8 + 8x^7}{x^{16}\left(\frac{1}{x^8} + \frac{1}{x^7} + 1\right)^2}dx$$

$$= \int \frac{\frac{7}{x^8} + \frac{8}{x^9}}{\left(\frac{1}{x^8} + \frac{1}{x^7} + 1\right)^2}dx$$

Let $1 + \frac{1}{x^7} + \frac{1}{x^8} = t$

$\Rightarrow$ $\left(-\frac{7}{x^8} - \frac{8}{x^9}\right)dx = dt$

$\Rightarrow$ $\left(\frac{7}{x^8} + \frac{8}{x^9}\right)dx = -dt$

$\therefore$ $I = \int -\frac{dt}{t^2} = \frac{1}{t}$

$$= \frac{1}{1 + \frac{1}{x^7} + \frac{1}{x^8}} = \frac{x^8}{x^8 + x + 1} + k$$

Sol.3 (C)

Let $I = \int \frac{1 + \log x}{3 + x\log x}dx$

Put $3 + x\log x = t$

$\Rightarrow$ $(\log x + 1)\,dx = dt$

$$= \int \frac{1}{t}dt$$

$= \log|t| + c$

$= \log|3 + x\log x| + c$

or $\log\left|\frac{3}{2} + \frac{x\log x}{2}\right| + c$

Sol.4 (A)

We try to write $e^x\left(\frac{x^3 - 3x^2 + 5x - 1}{(x^2 + 1)^2}\right)$

$= e^x(f(x) + f'(x))$ for true suitable function f.

Let $f(x) = \frac{ax + b}{(x^2 + 1)}$

$\therefore$ $f(x) + f'(x)$

$= \frac{ax+b}{x^2+1} + \frac{(x^2+1)a-(ax+b)2x}{(x^2+1)^2}$

$= \frac{ax^3+(b-a)x^2+(a-2b)x+a+b}{(x^2+1)^2}$

Comparing with the given function, we have

$a = 1,\ b - a = -3,\ a - 2b = 5$

and $a + b = -1$

$\therefore$ $a = 1,\ b = -2$

Thus $\int (f(x)+f'(x))e^x\,dx = e^x f(x) + k$

Hence $\int \frac{x^3-3x^2+5x-1}{(x^2+1)^2}\,dx = e^x \cdot f(x)$

$= e^x \cdot \frac{x-2}{x^2+1} + k$ from (1)

$f(x) = \frac{x-2}{(x^2+1)}$

Sol.5 (B)

$\int \left(\frac{\sec^2 x - 7}{\sin^7 x}\right) dx = \int \left(\frac{\sec^2 x}{\sin^7 x} - \frac{7}{\sin^7 x}\right) dx$

$= \int \left(\frac{\sec^2 x}{\sin^7} - \frac{7\cos x \tan x}{\sin^8 x}\right) dx$

$= \int (\sin x)^{-7} \sec^2 x\,dx - \int \frac{7\cos x\tan x}{\sin^8 x}dx$

$= (\sin x)^{-7} \tan x$

$- \int -7(\sin x)^{-8} \tan x \cos x\,dx$

$- \int \frac{7\cos x\tan x}{\sin^8 x}\,dx$

$= \frac{\tan x}{(\sin x)^7} + c$

Sol.6 (A)

Let $\log x = t$

$\Rightarrow$ $e^t = x$ $\Rightarrow$ $e^t\,dt = dx$

$\therefore$ $\int (\sin t + \cos t)\,e^t\,dt$

$= \int \sin t\ e^t\,dt + \int \cos t \cdot e^t\,dt$

Apply by parts for 1st integration

$= (\sin t)\,e^t - \int e^t \cos t\,dt$

$+ \int \cos t\ e^t\,dt$

$= (\sin t)\,e^t + c$

$= \sin(\log x) \cdot x + c$

Sol.7 (C)

Let $x^x = t$

$\Rightarrow$ $x^x(1 + \log x)\,dx = dt$

$\therefore$ $\int 1.dt = t + k = x^x + k$

Sol.8 (D)

$I = \int \frac{dx}{x^2+2x+2} = \int \frac{dx}{x^2+2x+1+1}$

$= \int \frac{dx}{(x+1)^2+1^2} = \tan^{-1}(x+1) + c$

Sol.9 (C)

Let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow$ $dx = 2\sqrt{x} \cdot dx = 2t\,dt$

$\therefore$ $I = \int 2t^2 e^t\,dt$

$= 2\left[t^2.\int e^t\,dt - \int \left\{\frac{dt^2}{dt}.\int e^t\,dt\right\}dt\right]$

$= 2\left[t^2e^t - \int 2te^t\,dt\right]$

$= 2\left[t^2.e^t - 2t\int e^t.dt + 2\int \left\{\frac{dt}{dt}.\int e^t\,dt\right\}dt\right]$

$= 2[t^2.e^t - 2te^t + 2e^t] + c$

$= (2t^2 - 4t + 4)\,e^t + c$

$= \left(2x - 4\sqrt{x} + 4\right) e^{\sqrt{x}} + c$