Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,–1). (All questions have only one option correct).

Statement : In the argand plane z_1, z_2 and z_3 are respectively the vertices of an isosceles triangle ABC with AC = BC and equal angle θ. If z_4 is the incentre of triangle, then

Q.1 The value of $\left(\frac{AB}{IA}\right)^2 . \left(\frac{AC}{AB}\right)$ is :

(A) $\frac{(z_2 - z_1)(z_1 - z_3)}{(z_4 - z_1)^2}$ (B) $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)}$

(C) $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2}$ (D) None of these

Q.2 The value of $(z^4 - z_1)^2 (1 + \cos\theta) \sec\theta$ is :

(A) $(z_2 - z_1)(z_3 - z_1)$ (B) $\frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)}$

(C) $(z_2 - z_1)^2 (z_3 - z_1)$ (D) $(z_2 - z_1)(z_3 - z_1)^2$

Q.3 The value of $\frac{(CD)(ID)}{(AD)^2}$ is :

(A) $\frac{(z_1 + z_2 + 2z_3)}{(z_2 - z_1)^2}$

(B) $\frac{(z_1 + z_2 - z_3)(z_1 + z_2 - 2z_4)}{(z_2 - z_1)^2}$

(C) $\frac{(z_1 - z_2 + z_3)(z_1 + z_2 - 2z_4)}{(z_2 - z_1)^2}$

(D) None of these

Q.4 The value of $(z_2 - z_1)^2 \tan\theta . \tan\theta/2$ is

(A) $(z_1 + z_2 - 2z_3)(z_1 + z_2 - 2z_4)$

(B) $(z_1 + z_2 - z_3)(z_1 + z_2 - z_4)$

(C) $-(z_1 + z_2 - 2z_3)(z_1 + z_2 - 2z_4)$

(D) None of these

Statement : In the equation $z^2 + 2\lambda z + 1 = 0$, λ is a parameter which can take any real value, then

Q.5 The roots of this equation lie on a certain circle, if :

(A) $-1 < \lambda < 1$ (B) $\lambda > 1$

(C) $\lambda < 1$ (D) None of these

Q.6 One root lies inside the unit circle and one outside, if :

(A) $-1 < \lambda < 1$ (B) $\lambda > 1$

(C) $\lambda < 1$ (D) None of these

Q.7 For every large value of λ, the root are approximately :

(A) $-2\lambda, \frac{1}{\lambda}$ (B) $-\lambda, -\frac{1}{\lambda}$

(C) $-2\lambda, -\frac{1}{2\lambda}$ (D) None

Q.8 For complex nos. $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$. Then for all complex nos. z with ln z, we have

(A) $\frac{1+z}{1-z} \cap 0$ (B) $\frac{1-z}{1+z} \cap 0$

(C) $\frac{1-z}{1+z} \cap 1$ (D) None

Q.9 The complex number $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lie on

(A) the x-axis

(B) the straight line y = 5

(C) a circle passing through the origin

(D) None

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 2) (COMPLEX NUMBER) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

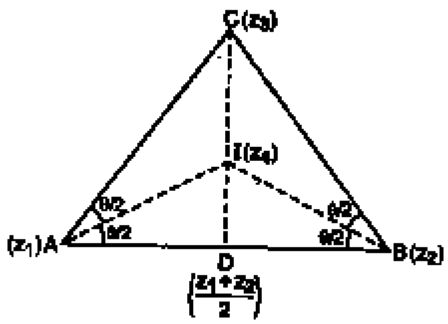
Que.	1	2	3	4	5	6	7	8	9
Ans.	C	A	B	C	A	B	C	B	A

SOLUTIONS

Sol.1 - 4

1 - (C), 2 - (A), 3 - (B), 4 - (C)

Let I be the incentre of ΔABC. Then,



$$\angle IAB = \frac{\theta}{2} \text{ and } \angle IAC = \frac{\theta}{2}$$

$$\Rightarrow \frac{z_2 - z_1}{|z_2 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|} e^{-i\theta/2}$$

$$\text{and } \frac{z_3 - z_1}{|z_3 - z_1|} = \frac{z_4 - z_1}{|z_4 - z_1|} e^{i\theta/2}$$

$$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{|z_2 - z_1||z_3 - z_1|} = \frac{(z_4 - z_1)^2}{|z_4 - z_1|^2} e^0$$

{on multiplying above equations}

$$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = \frac{AB.AC}{(IA)^2}$$

$$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = \left(\frac{AB}{IA}\right)^2 . \frac{AC}{AB} \quad ...(i)$$

$$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = 2\left(\frac{AD}{IA}\right)^2 \left(\frac{AC}{AD}\right)$$

$\{\because AB = 2AD\}$

$$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = 2\left(\cos^2 \frac{\theta}{2}\right)\left(\frac{1}{\cos\theta}\right)$$

$\therefore (z_4 - z_1)^2 (1 + \cos\theta). \sec\theta$

$= (z_2 - z_1)(z_3 - z_1) \quad ....(ii)$

Again, we have $AB \perp CD$

$$\therefore \frac{\frac{z_1 + z_2}{2} - z_3}{\left|\frac{z_1 + z_2}{2} - z_3\right|} = \frac{z_2 - z_1}{|z_2 - z_1|} e^{i\pi/2}$$

also $\angle IDB = \frac{\pi}{2}$

$$\therefore \frac{\frac{z_1 + z_4}{2} - z_4}{\left|\frac{z_1 + z_4}{2} - z_4\right|} = \frac{z_2 - z_1}{|z_2 - z_1|} e^{i\pi/2}$$

multiplying above equations,

$$\frac{z_1 + z_2 - 2z_3}{2CD} . \frac{z_1 + z_2 - 2z_4}{2ID} = -\frac{(z_2 - z_1)^2}{AB^2}$$

$$\Rightarrow -\frac{(z_1 + z_2 - 2z_3).(z_1 + z_2 - 2z_4)}{(z_2 - z_1)^2} = \frac{4(CD.ID)}{4AD^2}$$

$\{\because AB = 2AD\}$

$$\text{or } -\frac{(z_1 + z_2 - 2z_3).(z_1 + z_2 - 2z_4)}{(z_2 - z_1)^2} = \frac{CD}{AD} . \frac{ID}{AD}$$

$$\text{or } -\frac{(z_1 + z_2 - 2z_3)(z_1 + z_2 - 2z_4)}{(z_2 - z_1)^2} = \tan\theta . \tan\frac{\theta}{2}$$

or $(z_2 - z_1)^2 \tan\theta . \tan\frac{\theta}{2}$

$= -(z_1 + z_2 - 2z_3)(z_1 + z_2 - 2z_4)$

Sol.5 - 7

5 - (A), 6 - (B), 7 - (C)

We have, $z^2 + 2\lambda z + 1 = 0$

$$\Rightarrow z = \frac{-2\lambda \pm \sqrt{4\lambda^2 - 4}}{2}$$

$$\Rightarrow z = -\lambda \pm \sqrt{\lambda^2 - 1} \quad ....(i)$$

Case I : When $-1 < \lambda < 1$

If $-1 < \lambda < 1$, we have

$\lambda^2 < 1$

$\Rightarrow \lambda^2 - 1 < 0$

$\Rightarrow \lambda^2 - 1 = -u^2$, where $u > 0$

$\therefore \sqrt{\lambda^2 - 1} = \pm iu$

$\therefore z = -\lambda \pm iu$

$\therefore |z + \lambda| = u$

$\Rightarrow$ z lies on circle centre $(-\lambda + 0i)$ and radius u.

Case II : When $\lambda > 1$

If $\lambda > 1$, then

$\lambda^2 > 1$

$\Rightarrow$ $\lambda^2 - 1 > 0$

$\Rightarrow$ $\lambda^2 - 1 = u^2$, where $u \in R$

$\therefore$ $\sqrt{\lambda^2 - 1} = \pm u$

Let $z_1 = -\lambda + u$ and $z_2 = -\lambda - u$. Then,

$z_1 z_2 = 1 \Rightarrow |z_1 z_2| = 1$

$\Rightarrow$ $|z_1||z_2| = 1$

$\Rightarrow$ either ($|z_1| < 1$ and $|z_2| > 1$)

or ($|z_1| > 1$ and $|z_2| < 1$)

$\Rightarrow$ one root lies inside $|z| < 1$ and the outside $|z| < 1$.

Case III : When λ is very large :

In this case, roots of the equation are

$z_1 = -\lambda + u$ and $z_2 = -\lambda - u$

Also z_1 and z_2 are real numbers.

We have,

$\lambda^2 - 1 = u^2$

$\therefore$ $\sqrt{\lambda^2 - 1} = \pm u$

Since λ is very large

$\Rightarrow$ u is also large and $u \approx \lambda$

Hence, the roots are:

$$z_1 = -\lambda + u = \frac{u^2 - \lambda^2}{u + \lambda} = -\frac{1}{2\lambda}$$

Sol.8 **(B)**

Let $z = x + iy$. Since $1 \cap z$, we get $1 \leq x$ and $0 \leq y$.

Now, $$\frac{1-z}{1+z} = \frac{(1-x) - iy}{1 + x + iy}$$

$$= \frac{[(1-x) - iy][(1+x) - iy]}{(1+x)^2 + y^2}$$

$$= \frac{1 - x^2 - y^2 - 2iy}{(1+x)^2 + y^2}$$

Since $$\frac{1 - x^2 - y^2}{(1+x)^2 + y^2} \leq 0$$

$$\frac{-2y}{(1+x)^2 + y^2} \leq 0$$

we get $\frac{1-z}{1+z} \cap 0$.

Sol.9 **(A)**

$$\left|\frac{z - 5i}{z + 5i}\right| = 1$$

$\Rightarrow$ $|z - 5i| = |z - (-5i)|$

$\Rightarrow$ z is equidistant from $-5i$ and $5i$

$\Rightarrow$ z lies on the real axis.