Dear student following is a Moderate level [O ● O] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-7 (+3, –1). (All questions have only one option correct)

Let $P = \int e^{ax} \cos bx \, dx$, $Q = \int e^{ax} \sin bx \, dx$

Here P and Q are the function of x neglecting integration constant. This type of integration is done by complex number method, in which we assume P and Q as real and imaginary part of a function respectively. So, then integrate as a whole and after easy integration , we break up into real and imaginary part as such,

$$P + iQ = \int e^{ax} \cdot e^{ibx} \, dx$$

$$= \int e^{(a+ib)x} \, dx = \frac{e^{(a+ib)x}}{a+ib}$$

Q.1 The value of $(P^2 + Q^2)(a^2 + b^2)$ is

(A) e^{ax} (B) e^{2ax}

(C) $e^{(a^2+b^2)x}$ (D) $e^{2ab\,x}$

Q.2 The value of tan bx is

(A) $\frac{bP+Qa}{aP-bQ}$ (B) $\frac{bP-Qa}{aP+Qb}$

(C) $\frac{aP-bQ}{bP-Qa}$ (D) $\frac{aP+bQ}{bP-Qa}$

Q.3 The value of $\tan\left(bx + \frac{\pi}{4} - \tan^{-1}\left(\frac{b}{a}\right)\right)$ is

(A) $\frac{Q+P}{Q-P}$ (B) $\frac{Q+P}{P-Q}$ (C) $\frac{Q-P}{P+Q}$ (D) $\frac{-Q+P}{P+Q}$

If a function $f : R \to R$ is defined as

(i) f(x) is a differentiable and integrable function $\forall$ all $x \in R$

(ii) f(x) satisfies the property

$$\int f(x) \sin x . \cos x \, dx = \frac{1}{2(b^2 - a^2)} \ln f(x) + c$$

(iii) $f(x) > 0 \ \forall \ x \in R$.

(iv) assume $|a| < |b|$

Now let $\int f(x)\,dx = H(x) + c'$

Q.4 Then f(x) is

(A) $\frac{1}{a \sin x + b \cos x}$ (B) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$

(C) $\frac{1}{a^2 \sin x + b^2 \cos x}$ (D) $\frac{1}{a \sin^2 x + b \cos^2 x}$

Q.5 The range of function y = f(x) is

(A) $\left[\frac{1}{a}, \frac{1}{b}\right]$ (B) $\left(\frac{1}{a^2}, \frac{1}{b^2}\right)$

(C) $\left[\frac{1}{a^2}, \frac{1}{b^2}\right]$ (D) $\left[\frac{1}{a^2+1}, \frac{1}{b^2+1}\right]$

Q.6 What is the function H(x) ?

(A) $\frac{1}{ab}\tan^{-1}$ (b/a tan x)

(B) $\frac{1}{ab}\tan^{-1}\left(\frac{a}{b} \tan x\right)$

(C) $\frac{a}{b}\tan^{-1}\left(\frac{b}{a} \tan x\right)$

(D) $\frac{b}{a}\tan^{-1}\left(\frac{a}{b} \tan x\right)$

Q.7 What is the range of function y = H (x) ?

(A) $\left[\frac{-1}{ab}, \frac{1}{ab}\right]$ (B) $[0, \infty)$

(C) R (D) $\left(\frac{-1}{ab}, \frac{1}{ab}\right)$

MATHEMATICS IIT JEE (SEPT. 4th WEEK CLASS TEST 3) (INDEFINITE INTEGRATION) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○					
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	B	A	B	B	C	B	C

SOLUTIONS

Sol.1 to 3 (B, A, B)

Let $P = \int e^{ax} \cos bx\, dx$

and $Q = \int e^{ax} \sin bx\, dx$

$\therefore \quad P + iQ = \int e^{ax} (\cos bx + i \sin bx)\, dx$

[We may apply integration by parts twice]

$= \int e^{ax} . e^{i\,bx}\, dx = \int e^{(a+ib)x}\, dx$

$= \dfrac{e^{(a+ib)x}}{(a+ib)} + c$

$= \dfrac{e^{ax}(\cos bx + i \sin bx)(a - ib)}{a^2 + b^2} + c$

$= \dfrac{e^{ax}\{(a \cos bx + b \sin bx)\} + i\, e^{ax}\{(a \sin bx - b \cos bx)}{(a^2 + b^2)} + c$

$= \dfrac{e^{ax}(a \cos bx + b \sin bx)}{(a^2 + b^2)} + i\dfrac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + c$

Equating real and imaginary parts on both sides, we get

$P = \int e^{ax} \cos bx\, dx$

$= \dfrac{e^{ax}(a \cos bx + b \sin bx)}{(a^2 + b^2)} + c$

$= \dfrac{1}{r} e^{ax} \cos (bx - \phi) + c$

and $Q = \int e^{ax} \sin bx\, dx$

$= \dfrac{e^{ax}(a \sin bx - b \cos bx)}{(a^2 + b^2)} + c$

$= \dfrac{1}{r} e^{ax} \sin (bx - \phi) + c$

where $r = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}\left(\dfrac{b}{a}\right)$

$\therefore \quad (P^2 + Q^2)\, r^2 = e^{2ax}$

(neglecting constant of integration)

$\therefore \quad (P^2 + Q^2)(a^2 + b^2) = e^{2ax}$

Sol.4 to 7 (B, C, B, C)

Given $\int f(x) \sin x \cos x\, dx$

$= \dfrac{1}{2(b^2 - a^2)} \ln f(x) + c$

Differentiating both sides w.r.t x then

$f(x) \sin x \cos x = \dfrac{1}{2(b^2 - a^2)} \cdot \dfrac{f'(x)}{f(x)}$

$\Rightarrow \quad 2(b^2 - a^2) \sin x \cos x = \dfrac{f'(x)}{\{f(x)\}^2}$

$\Rightarrow \quad 2b^2 \sin x \cos x - 2a^2 \sin x \cos x = \dfrac{f'(x)}{\{f(x)\}^2}$

Integrating both sides w.r.t x, we get

$- b^2 \cos^2 x - a^2 \sin^2 x = - \dfrac{1}{f(x)}$

or $\quad f(x) = \dfrac{1}{(a^2 \sin^2 x + b^2 \cos^2 x)}$

$H(x) = \int \dfrac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$

$= \dfrac{1}{ab} \tan^{-1}\left(\dfrac{a}{b} \tan x\right)$