Dear student following is a Moderate level [○○●○○] test paper. Score of 9 Marks in 10 Minutes would be a satisfactory performance. Questions 1-6 (+3, –1). (All questions have one option correct only.)

Q.1 If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5 =$

(A) 0 (B) 25
(C) 625 (D) None of these

Q.2 If $A_i = \begin{bmatrix} a^i & b^i \\ b^i & a^i \end{bmatrix}$ and if $|a| < 1$, $|b| < 1$, then $\sum_{i=1}^{\infty} \det(A_i)$ is equal to-

(A) $\frac{a^2}{(1-a)^2} - \frac{b^2}{(1-b)^2}$ (B) $\frac{a^2 - b^2}{(1-a^2)(1-b^2)}$

(C) $\frac{a^2}{(1-a)^2} + \frac{b^2}{(1-b)^2}$ (D) $\frac{a^2}{(1+a)^2} - \frac{b^2}{(1+b)^2}$

Q.3 If α, β are different from 1 and are the roots of $ax^3 + bx^2 + cx + d = 0$ and $(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = \frac{25}{2}$, then the determinant $\Delta = \begin{vmatrix} \frac{\alpha}{1-\alpha} & \frac{\beta}{1-\beta} & \frac{\gamma}{1-\gamma} \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}$ equals-

(A) $\frac{25d}{2a}$ (B) $\frac{25d}{a}$

(C) $\frac{-25d}{a+b+c+d}$ (D) None of these

Q.4 If a, b, c are distinct, and $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$ then $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$ vanishes if-

(A) $x = \frac{1}{3}(a+b+c)$ (B) $x = \frac{2}{3}(a+b+c)$

(C) $x = a + b + c$ (D) None of these

Q.5 Let

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec}^2 x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$$

then value of $\int_{\pi/4}^{\pi/2} f(x)\,dx$ is-

(A) 0 (B) $\frac{\pi}{48}$

(C) $-\frac{\pi}{2} - \frac{\pi}{15\sqrt{2}}$ (D) None of these

Q.6 If $x \in R$ and $n \in I$, then the determinant $\Delta = \begin{vmatrix} \sin(n\pi) & \sin x - \cos x & \log \tan x \\ \cos x - \sin x & \cos[(2n+1)\pi/2] & \log \cot x \\ \log \cot x & \log \tan x & \tan(n\pi) \end{vmatrix}$ equals-

(A) 0
(B) log tan x – log cot x
(C) tan (π /4 – x) (D) None of these

MATHEMATICS IIT JEE (06 / 06 / 2007) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	3	○	○	○	○	5	○	○	○	○
2	○	○	○	○	4	○	○	○	○	6	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6
Ans.	D	B	D	A	D	A

SOLUTIONS

Sol.1 (D)

$$D_1 = \begin{vmatrix} 1 & 15 & 8 \\ 1 & 35 & 9 \\ 1 & 25 & 10 \end{vmatrix}, \quad D_2 = \begin{vmatrix} 2 & 15 & 8 \\ 4 & 35 & 9 \\ 8 & 25 & 10 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} 3 & 15 & 8 \\ 9 & 35 & 9 \\ 27 & 25 & 10 \end{vmatrix}, \quad D_4 = \begin{vmatrix} 4 & 15 & 8 \\ 16 & 35 & 9 \\ 64 & 25 & 10 \end{vmatrix}$$

$$D_5 = \begin{vmatrix} 5 & 15 & 8 \\ 25 & 35 & 9 \\ 125 & 25 & 10 \end{vmatrix}$$

$\Rightarrow D_1 + D_2 + D_3 + D_4 + D_5$

$$= \begin{vmatrix} 15 & 75 & 40 \\ 55 & 175 & 45 \\ 225 & 125 & 50 \end{vmatrix}$$

$= 15(3125) - 75(-7375) + 40(-32500)$

$= 46875 + 553125 - 1300000$

$= -7000000$

Sol.2 (B)

$$|A_i| = \begin{vmatrix} a^i & b^i \\ b^i & a^i \end{vmatrix}$$

$= (a^i)^2 - (b^i)^2, \ |a| < 1, \ |b| < 1$

$$\sum_{i=1}^{\infty} |A_i| = (a^2 - b^2) + (a^4 - b^4) + (a^6 - b^6) + \ldots\ldots\ldots\ldots$$

$$= (a^2 + a^4 + a^6 + \ldots.) - (b^2 + b^4 + b^6 + \ldots\ldots\ldots)$$

$$= \frac{a^2}{1-a^2} - \frac{b^2}{1-b^2}$$

$$= \frac{a^2 - a^2b^2 - b^2 + a^2b^2}{(1-a^2)(1-b^2)}$$

$$= \frac{a^2 - b^2}{(1-a^2)(1-b^2)}$$

Sol.3 (D)

Taking α, β, γ common from C_1, C_2, C_3 respectively, we get

$$\Delta = \alpha\beta\gamma \begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} & \frac{1}{1-\gamma} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix}$$

Apply $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= \alpha\beta\gamma \begin{vmatrix} \frac{1}{1-\alpha} & \frac{1}{1-\beta} - \frac{1}{1-\alpha} & \frac{1}{1-\gamma} - \frac{1}{1-\alpha} \\ 1 & 0 & 0 \\ \alpha & \beta-\alpha & \gamma-\alpha \end{vmatrix}$$

$$= \frac{\alpha\beta\gamma(-1)(\beta-\alpha)(\gamma-\alpha)}{(1-\alpha)(1-\beta)(1-\gamma)} \begin{vmatrix} 1-\gamma & 1-\beta \\ 1 & 1 \end{vmatrix}$$

$$= \frac{\alpha\beta\gamma(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}{(1-\alpha)(1-\beta)(1-\gamma)}$$

As α, β, γ are the roots of

$ax^3 + bx^2 + cx + d = 0$,

$ax^3 + bx^2 + cx + d$

$= a(x - \alpha)(x - \beta)(x - \gamma)$

and $\alpha\beta\gamma = -d/a$

Thus, $\Delta = \dfrac{(-d/a)(25/2)}{(a+b+c+d)/a}$

$$= -\frac{25d}{2(a+b+c+d)}$$

Sol.4 (A)

Multiplying C_1 by $(x - a)$, C_2 by $(x - b)$ and C_3 by $(x - c)$ we get

$$\Delta = \frac{1}{ABC} \begin{vmatrix} A & B & C \\ A^3 & B^3 & C^3 \\ ABC & ABC & ABC \end{vmatrix}$$

where $A = x - a$, $B = x - b$, $C = x - c$

$$\Delta = \begin{vmatrix} A & B & C \\ A^3 & B^3 & C^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (-1)(-1) \begin{vmatrix} 1 & 1 & 1 \\ A & B & C \\ A^3 & B^3 & C^3 \end{vmatrix}$$

$= (B - C)(C - A)(A - B)(A + B + C)$

$= (c - b)(a - b)(b - a)$
$[3x - (a + b + c)]$

Note that Δ become 0 when

$$x = \frac{1}{3}(a + b + c)$$

Sol.5 (D)

Applying $R_2 \to R_2 - R_3$, we get

$$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec}^2 x \\ -\sin^2 x & 0 & 0 \\ 1 & \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$$

$$= -(-\sin^2 x) \begin{vmatrix} \cos x & \sec^2 x + \cot x \operatorname{cosec}^2 x \\ \cos^2 x & \operatorname{cosec}^2 x \end{vmatrix}$$

$= \sin^2 x\, [\cos x \operatorname{cosec}^2 x$
$- \cos^2 x (\sec^2 x + \cot x \operatorname{cosec}^2 x)]$

$$= \cos x - \sin^2 x - \frac{\cos^3 x}{\sin x}$$

$$= \cos x - \frac{1}{2}(1 - \cos 2x)$$

$$- \left(\frac{1}{\sin x} - \sin x\right) \cos x$$

Thus, $\int_{\pi/4}^{\pi/2} f(x)\,dx = \int_{\pi/4}^{\pi/2} \cos x\,dx - \frac{1}{2}\int_{\pi/4}^{\pi/2} dx$

$$+ \frac{1}{2}\int_{\pi/4}^{\pi/2} \cos 2x\,dx$$

$$- \int_{\pi/4}^{\pi/2} \left(\frac{1}{\sin x} - \sin x\right) \cos x\,dx$$

$$= 1 - \frac{1}{\sqrt{2}} - \frac{1}{2}\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + \frac{1}{4}(0 - 1)$$

$$- \left[\left(\log|t| - \frac{t^2}{2}\right)\right]_{1/\sqrt{2}}^{1} \text{ where } t = \sin x$$

$$= 1 - \frac{1}{\sqrt{2}} - \frac{\pi}{8} - \frac{1}{2}\log 2$$

Sol.6 (A)

We can write Δ as

$$\Delta = \begin{vmatrix} 0 & \sin x - \cos x & \log\tan x \\ -(\sin x - \cos x) & 0 & -\log\tan x \\ -\log\tan x & \log\tan x & 0 \end{vmatrix}$$

$$= (-1)^3 \begin{vmatrix} 0 & -(\sin x - \cos x) & -\log\tan x \\ \sin x - \cos x & 0 & \log\tan x \\ \log\tan x & -\log\tan x & 0 \end{vmatrix}$$

[taking –1 common from R_1, R_2 and R_3]

$= -\Delta$ [using reflection property]

$\Rightarrow 2\Delta = 0 \quad \Rightarrow \Delta = 0.$