Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 12 Marks in 10 Minutes would be a satisfactory performance. Questions 1-7 (+3, –1). (All questions have one option correct only.)

Q.1 If the entries in a 3 × 3 determinant are either 0 or 1 then the greatest value of this determinant is-

(A) 1 (B) 2
(C) 3 (D) 9

Q.2 If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ =

(A) 2AB (B) 2BA
(C) A + B (D) AB

Q.3 If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then-

(A) $\Delta_1 = 3(\Delta_2)^2$ (B) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(C) $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$ (D) $\Delta_1 = 3\Delta_2^{3/2}$

Q.4 A root of the equation

$$\Delta = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0 \text{ is-}$$

(A) $\frac{1}{2}(a + b + c)$ (B) 0
(C) –1 (D) 1

Q.5 If $A = \begin{bmatrix} x & 3 & 2 \\ -3 & y & -7 \\ -2 & 7 & 0 \end{bmatrix}$ and $A = -A^T$,

then x + y is equal to-

(A) 2 (B) –1
(C) 0 (D) 12

Q.6 Let a, b, c be positive real numbers. The following system of equations in x, y and z

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1,$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ has-}$$

(A) No solution
(B) Unique solution
(C) Infinitely many solutions
(D) Finitely many solutions

Q.7 The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval}$$

$-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is-

(A) 0 (B) 2
(C) 1 (D) 3

MATHEMATICS IIT JEE (07 / 06 / 2007) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	3	○	○	○	○	5	○	○	○	○
2	○	○	○	○	4	○	○	○	○	6	○	○	○	○
										7	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	B	C	B	B	C	B	C

SOLUTIONS

Sol.1 (B)

Let the entries of 3 × 3 determinant be a_1, a_2, a_3, b_1, b_2, b_3 and c_1, c_2, c_3

i.e. $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= a_1b_2c_3 - a_1b_3c_2 + a_2b_3c_1 - a_2b_1c_3 + a_3b_1c_2 - a_3b_2c_1$

$= [a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2] - [a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1]$

Now, given that each entries of 3 × 3 determinant are either 0 or 1. Therefore the value of determinant is maximum when value of each term in 1st bracket is 1 and value of each term in 2nd bracket is zero, i.e. value of determinant cannot exceed 3.

$\Rightarrow \quad a_1b_2c_3 = a_2b_3c_1 = a_3b_1c_2 = 1$ ....(1)

But when (1) holds value of determinant is zero.

So, let $\Delta = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2.$

Sol.2 (C)

Given AB = B and BA = A

$\therefore \quad A^2 + B^2 = A.A + B.B$

$= A(BA) + B(AB)$

$= (AB)A + (BA)B = (BA) + AB$

$= A + B$

Sol.3 (B)

$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$

$\therefore \quad \frac{d}{dx}\Delta_1 = \begin{vmatrix} 1 & b & b \\ 0 & x & b \\ 0 & a & x \end{vmatrix} + \begin{vmatrix} x & 0 & b \\ a & 1 & b \\ a & 0 & x \end{vmatrix} + \begin{vmatrix} x & b & 0 \\ a & x & 0 \\ a & a & 1 \end{vmatrix}$

$= \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix}$

$= 3\begin{vmatrix} x & b \\ a & x \end{vmatrix} = 3\Delta_2.$

Sol.4 (B)

Let x = 0, then given Δ reduces to

$\begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$

which is skew symmetric determinant of odd order.

i.e. $\Delta = 0$

$\Rightarrow$ x = 0 is one of the root of the equation

Sol.5 (C)

Given $A = -A^T$

$\Leftrightarrow$ A is a skew symmetric matrix

$\Leftrightarrow$ All diagonal entries are zero

$\Leftrightarrow$ x = 0, y = 0

$\Leftrightarrow$ x + y = 0

Sol.6 (B)

Let $\frac{x^2}{a^2} = X$, $\frac{y^2}{b^2} = Y$ and $\frac{z^2}{c^2} = Z$, then the given system of equations is $X + Y - Z = 1$, $X - Y + Z = 1$, $-X + Y + Z = 1$

The coefficient matrix is $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Clearly $|A| \neq 0$. So the given system of equations has unique solution.

Sol.7 (C)

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad (2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$$

Apply $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

or $(2\cos x + \sin x)$

$$\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$$

or $\quad (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore \quad \tan x = -2, 1.$

But $\tan x \neq -2$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\therefore \quad \tan x = 1$. So, $x = \frac{\pi}{4}$.