Dear student following is an Easy level [○ ● ○ ○ ○] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (All questions have one option correct only.)

Q.1 The matrix $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent of index-

(A) 2 (B) 3
(C) 4 (D) 6

Q.2 If A is a 3 × 3 matrix and B is its adjoint matrix. If |B| = 64, then |A| =

(A) ±2 (B) ±4
(C) ±8 (D) ±12

Q.3 If $A \neq A^2 = I$, then det. (I + A) =

(A) 0 (B) –1
(C) 1 (D) 2

Q.4 If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in N$, then A^{4n} equals-

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

(C) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q.5 If A is a square matrix, then $adj\ (A^T) - (adj\ A)^T$ is equal to-

(A) 2|A| (B) 2|A|I
(C) Null matrix (D) Unit matrix

Q.6 If A is a square matrix such that |A| = 2. Then for any +ve integer n, $|A^n|$ is equal to-

(A) 0 (B) 2n
(C) 2^n (D) n^2

Q.7 If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c each greater than zero along with $A^T A = I$, then the value of $\sum a^3$ equals-

(A) 0 (B) 1
(C) 4 (D) 3

Q.8 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$, where ω is a complex cube root of unity then adj A equals-

(A) $(\omega^2 - \omega)\overline{A}$ (B) $(\omega - \omega^2)\overline{A}$

(C) $\dfrac{(\omega^2 - \omega)\overline{A}}{3}$ (D) None of these

MATHEMATICS IIT JEE (08 / 06 / 2007) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	A	C	A	A	C	C	C	B

SOLUTIONS

Sol.1 (A)

$$A^2 = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore$ A is nilpotent of index 2.

Sol.2 (C)

Given adj A = B

$\because$ A adj A = |A| I

$\Rightarrow$ AB = |A| I

$\Rightarrow$ $|AB| = |A|^3\ I$

$\Rightarrow$ $|A|\ |B| = |A|^3$

$\Rightarrow$ $|A|^2 = |B|$ $\Rightarrow$ $|A|^2 = 64$

$\Rightarrow$ $|A| = \pm 8$

Sol.3 (A)

Given $A \neq A^2 = I$

$\Rightarrow$ $A^2 - I = 0$

$\Rightarrow$ $(A - I)(A + I) = 0$

Now $(A + I)^{-1}$ exist if $|A + I| \neq 0$

$\Rightarrow$ $(A - I)(A + I)(A + I)^{-1}$

$= (A + I)^{-1} . 0$

$\Rightarrow$ $(A - I)\ I = 0$

$\Rightarrow$ $A = I$ but $A \neq I$

$\therefore$ $|A + I| = 0$

Sol.4 (A)

$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and so on $A^{4n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol.5 (C)

$\because$ $(\text{Adj } A)^T = \text{adj } (A^T)$

$\Rightarrow$ $\text{adj } A^T - \text{adj } A^T$ = zero matrix.

Sol.6 (C)

$|A^n|$ = |A A A .... n times|

= |A| |A| |A| .... n times

= 2.2.2 .... n times

$= 2^n$

Sol.7 (C)

Given $AA^T = I$

$\Rightarrow$ $|AA^T| = |I|$ is $|A|^2 = 1$

$\because$ $|A^T| = |A|$

and $|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

$\Rightarrow$ $a^3 + b^3 + c^3 - 3abc = 1$

(a, b, c > 0 $\because$ AM $\geq$ GM

$\Rightarrow$ $a^3 + b^3 + c^3 \geq 3abc$)

$\Rightarrow$ $\sum a^3 = 4$

Sol.8 (B)

$$\text{adj } A = \begin{bmatrix} \omega - \omega^2 & \omega - \omega^2 & \omega - \omega^2 \\ \omega - \omega^2 & \omega(\omega - \omega^2) & \omega^2(\omega - \omega^2) \\ \omega - \omega^2 & \omega^2(\omega - \omega^2) & \omega(\omega - \omega^2) \end{bmatrix}^T$$

$$= (\omega - \omega^2) \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}^T$$

$$= (\omega - \omega^2)(\overline{A})$$