Dear student following is an Easy level [O ● O O O] test paper. Score of 15 Marks in 10 Minutes would be a satisfactory performance. Questions 1-8 (+3, –1). (All questions have one option correct only.)

Q.1 $\begin{vmatrix} \log_z x & \log_z y & 1 \\ 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \end{vmatrix}$ is equal to-

(A) 3 (B) 1
(C) log x + log y + log z (D) 0

Q.2 If $a \neq p$, $b \neq q$, $c \neq r$ and $\Delta = \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ equals-

(A) 0 (B) 1
(C) –1 (D) 2

Q.3 Let A28, 3B9, 62C be divisible by a fixed integer k, where A, B and C are integers between 0 and 9. Then $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by-

(A) k (B) k^2
(C) k + 1 (D) None of these

Q.4 For a fixed positive integer n if

$$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$

then that $\left(\frac{D}{(n!)^3} - 4\right)$ is divisible by-

(A) n (B) n!
(C) n^2 (D) None of these

Q.5 The system of linear equations x + y + z = 6, x + 2y + 3z = 14 and 2x + 5y + λz = 6($\lambda \in$ R) will have a unique solution if-

(A) $\lambda = 8$ (B) $\lambda = 1$
(C) $\lambda \neq 8$ (D) $\lambda \neq 1$

Q.6 If (x, y) are the co-ordinates of a point in the plane, then $\begin{vmatrix} 3 & 4 & 2 \\ 5 & 8 & 2 \\ x & y & 2 \end{vmatrix} = 0$ represent-

(A) A striaght line || to y-axis
(B) A straight line || to x-axis
(C) A straight line
(D) A circle

Q.7 If a, b, c are in G.P., or (x – α) is a factor of $ax^2 + 2bx + c$ then value of the determinant

$$\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$$ is equal to-

(A) 1 (B) –1
(C) 2 (D) 0

Q.8 If the system of linear equations
x + 2ay + az = 0
x + 3by + bz = 0
x + 4cy + cz = 0
has a non-zero solution, then a, b, c
(A) Satisfy a + 2b + 3c = 0
(B) Are in A.P.
(C) Are in G.P.
(D) are In H.P.

MATHEMATICS IIT JEE (09 / 06 / 2007) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	D	D	A	A	C	C	D	D

SOLUTIONS

Sol.1 (D)

Given, determinant

$$= \begin{vmatrix} \log x/\log z & \log y/\log z & 1 \\ 1 & \log y/\log x & \log z/\log x \\ \log x/\log y & 1 & \log z/\log y \end{vmatrix}$$

Multiplying R_1 by log z, R_2 by log x and R_3 by log y, we obtain

$$= \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z}$$

$$(\log x \log y \log z) \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Sol.2 (D)

$$\Delta = \begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

we get $\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0$

On expanding w.r.to C_3 we get

$$c \begin{vmatrix} a-p & q-b \\ a-p & 0 \end{vmatrix} + (r - c)$$

$$\begin{vmatrix} p & b \\ a-p & q-b \end{vmatrix} = 0$$

$\Rightarrow$ $-c(a - p)(q - b) + (r - c)[p(q - b) - b(a - p)] = 0$

or $c(p - a)(q - b) + p(q - b)(r - c) + b(p - a)(r - c) = 0$

Dividing by $(p - a)(q - b)(r - c)$ we get

$$\frac{c}{r-c} + \frac{p}{p-a} + \frac{b}{q-b} = 0$$

$$\Rightarrow \frac{(c-r)+r}{(r-c)} + \frac{p}{p-a} + \frac{(b-q)+q}{q-b} = 0$$

or $-1 + \dfrac{r}{r-c} + \dfrac{p}{p-a} - 1 + \dfrac{q}{q-b} = 0$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2.$$

Sol.3 (A)

We have

A28 = 100 × A + 20 + 8

and 3B9 = 300 + B × 10 + 9

62C = 600 + 20 + C × 1

Since these are divisible by k then

A28 = n_1k 3B9 = n_2k 62C = n_3k

where $n_1, n_2, n_3 \in I$.

$$\Delta = \begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$$

Operating $R_2 \to R_2 + 100R_1 + 10R_3$

$$= \begin{vmatrix} A & 3 & 6 \\ A28 & 3B9 & 62C \\ 2 & B & 2 \end{vmatrix} = \begin{vmatrix} A & 3 & 6 \\ n_1k & n_2k & n_3k \\ 2 & B & 2 \end{vmatrix}$$

$$= k \begin{vmatrix} A & 3 & 6 \\ n_1 & n_2 & n_3 \\ 2 & B & 2 \end{vmatrix}$$

Which is divisible by k.

Sol.4 (A)

D = n!(n + 1)! (n + 2)!

$$\begin{vmatrix} 1 & (n+1) & (n+2)(n+1) \\ 1 & (n+2) & (n+3)(n+2) \\ 1 & (n+3) & (n+4)(n+3) \end{vmatrix}$$

By talking n!, (n + 1) and (n + 2)! Common from R_1, R_2 and R_3 respectively.

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$

$= (n!)^3 (n + 1)^2 (n + 2)$

$$\begin{vmatrix} 1 & n+1 & (n+2)(n+1) \\ 0 & 1 & 2(n+2) \\ 0 & 1 & 2(n+3) \end{vmatrix}$$

$\Rightarrow \frac{D}{(n!)^3}$

$= (n + 1)^2 (n + 2) [2(n + 3) - 2(n + 2)]$
$= 2(n + 1)^2 (n + 2)$

$\Rightarrow \frac{D}{(n!)^3} - 4 = 2[(n + 1)^2 (n + 2) - 2]$

or $\frac{D}{(n!)^3} - 4 = 2[n^3 + 4n^2 + 5n + 2 - 2]$

$= 2n [n^2 + 4n + 5]$

Thus $\frac{D}{(n!)^3} - 4$ is divisible by n.

Sol.5 (C)

The system of linear equations

$x + y + z = 6$
$x + 2y + 3z = 14$
$2x + 5y + \lambda z = 6$

will have a unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0 \text{ or if}$$

$1.(2\lambda - 15) - 1(\lambda - 6) + 1(5 - 4) \neq 0$

or if $\lambda - 8 \neq 0$ or $\lambda \neq 8$.

Sol.6 (C)

$$\begin{vmatrix} 3 & 4 & 2 \\ 5 & 8 & 2 \\ x & y & 2 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 2 \\ 2 & 4 & 0 \\ x-5 & y-8 & 0 \end{vmatrix}$$

$= 2(2y - 16 - 4x + 20)$
$= 2(2y - 4x + 4)$

$\therefore$ given determinant = 0

$\Rightarrow 2y - 4x + 4 = 0$

$\Rightarrow 2x - y - 2 = 0$ which represents straight line.

Sol.7 (D)

$C_1 \rightarrow C_1\alpha + C_2$

$$\begin{vmatrix} a\alpha+b & b & a\alpha+b \\ b\alpha+c & c & b\alpha+c \\ a\alpha^2+2b\alpha+c & b\alpha+c & 0 \end{vmatrix}$$

Since $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$, so $a\alpha^2 + 2b\alpha + c = 0$

$$= \begin{vmatrix} a\alpha+b & b & a\alpha+b \\ b\alpha+c & c & b\alpha+c \\ 0 & b\alpha+c & 0 \end{vmatrix} = 0$$

Sol.8 (D)

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \quad C_2 \rightarrow C_2 - 2C_3$$

$$\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \rightarrow R_3 - R_2,\ R_2 \rightarrow R_2 - R_1$$

$$\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore$ a, b, c are in Harmonic Progression.