Q.1 The value of n for which the determinant

$$\Delta = \begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix} = 0 \text{ is-}$$

(A) 2 (B) 3 (C) 4 (D) None

Q.2 Let $\Delta = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\theta) & -\sin(\alpha+\theta) & 1 \end{vmatrix}$, then-

(A) $\Delta \in [1-\sqrt{2}, 1+\sqrt{2}]$ (B) $\Delta \in [-1, 1]$

(C) $\Delta \in [-\sqrt{2}, \sqrt{2}]$ (D) None of these

Q.3 If the value of determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is +ve, then-

(A) $xyz > 1$ (B) $xyz > -8$

(C) $xyz < -8$ (D) $xyz > -2$

Q.4 In a triangle ABC if $\begin{vmatrix} 1 & a & b \\ 1 & b & c \\ 1 & c & a \end{vmatrix} = 0$, then the value of $\sin^2 A + \sin^2 B + \sin^2 C$ =

(A) 1 (B) $\frac{9}{4}$ (C) $\frac{4}{9}$ (D) $3\sqrt{3}$

Q.5 If a > b > c and the system of equations ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0 has a non trivial solution, then both the roots of the quadratic equation $at^2 + bt + c$ are-

(A) Real (B) Opposite sign

(C) +ve (D) Complex

Q.6 If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ then det [adj (adj A)]=

(A) 14^2 (B) 14^3

(C) 14^4 (D) None of these

Q.7 If $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then A^{50} is-

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

Q.8 Let $\{\Delta_1, \Delta_2, \Delta_3, \ldots\ldots \Delta_k\}$ be the set of third order determinants that can be made with the distinct non zero real numbers $a_1, a_2, a_3, \ldots\ldots a_9$, then-

(A) k = 9! (B) $\sum_{i=1}^{k} \Delta_i = 0$

(C) At least one $\Delta_i = 0$ (D) None of these

Q.9 $A + B = 2B^T$ and $3A + 2B = I$, where A and B are matrices of same order, then 5A + 2B equals-

(A) – I (B) – 2I

(C) I (D) 2I

Q.10 If $z = \begin{vmatrix} 1 & i & 7+8i \\ -i & 2 & 3+2i \\ 7-8i & 3-2i & 4 \end{vmatrix}$, then

(A) Re(z) = 0 (B) Im(z) = 0

(C) arg z = $\frac{\pi}{4}$ (D) Re(z)+Im(z)= 0

MATHEMATICS IIT JEE (CLASS TEST - 1) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○
										10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	B	B	A,B	C	D	A	D	B

SOLUTION

Sol.1 (C)

$\because \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore \quad$ Apply $R_1 \to R_1 + R_2$

$$\begin{vmatrix} {}^9C_4 & {}^{10}C_6 & {}^{11}C_8 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix} = 0$$

Now R_1 is identical to R_3 if

$n = 4$.

Sol.2 (A)

Apply $R_3 \to R_3 - R_1 \cos\theta + R_2 \sin\theta$

$$\Delta = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1+\sin\theta-\cos\theta \end{vmatrix}$$

expand along R_3

$\Delta = (1 + \sin\theta - \cos\theta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right)$

Now since $-1 \le \sin\left(\theta - \frac{\pi}{4}\right) \le 1$

$\Rightarrow \quad -\sqrt{2} \le \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) \le \sqrt{2}$

$\Rightarrow \quad 1-\sqrt{2} \le 1+\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right) \le 1+\sqrt{2}$

Thus $\Delta \in \left[1-\sqrt{2}, 1+\sqrt{2}\right]$

Sol.3 (B)

On simplifying the determinant we have

$\Delta = xyz - (x + y + z) + 2$

Given $\Delta > 0$

$\Rightarrow \quad xyz - (x + y + z) + 2 > 0$

$\Rightarrow \quad xyz + 2 > x + y + z$

$\because \quad$ A.M. > G.M.

$\Rightarrow \quad \frac{x+y+z}{3} > (xyz)^{1/3}$

$\Rightarrow \quad xyz + 2 > 3\,(xyz)^{1/3}$

Let $(xyz) = t^3$

$\Rightarrow \quad t^3 - 3t + 2 > 0$

$\Rightarrow \quad (t - 1)^2 (t + 2) > 0$

$\Rightarrow \quad t = -2$ or $t^3 > -8$

or $\quad xyz > -8$

Sol.4 (B)

On simplifying we have

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \quad \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]=0$

$\Rightarrow \quad a - b = 0,\ b - c = 0,\ c - a = 0$

$\Rightarrow \quad a = b = c$

Thus ΔABC is an equilateral triangle

$\Rightarrow \quad A = B = C = 60^\circ$

Thus $\sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 60^\circ + \sin^2 60^\circ + \sin^2 60$

$= \frac{9}{4}$

Sol.5 (B)

For non trivial solution $\Delta = 0$

$$\Rightarrow \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$\Rightarrow \quad -(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow \quad (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow \quad -\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$

$\Rightarrow \quad a + b + c = 0$ or $b = -(a + c)$

Now, $at^2 + bt + c = 0$

disc $= b^2 - 4ac = (a - c)^2 +$ ve

$\therefore \quad$ roots are real.

Now if roots are of opposite sign, then

$\alpha - \beta$ and $\beta - \alpha$ are +ve

Thus $(\alpha + \beta)^2 - 4\alpha\beta$ = +ve

or $\left(-\frac{b}{a}\right)^2 - \frac{4c}{a} = \frac{\text{disc}}{a^2}$ = +ve

$\Rightarrow$ Real of opposite sign.

Sol.6 (C)

Det A = 14

$\because$ adj (adj A) = $|A|^{n-2} A = |A|\ A$

$\therefore$ Det. [adj (adj A)] = Det (|A| A)

$\Rightarrow$ $|A|^3$ Det A = $|A|^4 = 14^4$

Sol.7 (D)

$$A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix},$$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix},$$

$$A^8 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$ and so on implies

$$A^n = \begin{bmatrix} 1 & 0 \\ n/2 & 1 \end{bmatrix}$$

$$\therefore \quad A^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

Sol.8 (A, B)

The number of third order determinants

= The number of arrangements of nine different numbers in nine places = 9!.

Corresponding to each determinant made, there is a determinant obtained by interchanging two consecutive rows (or columns)

So the sum of this pair will be 0.

$\therefore$ The sum of all the determinants

= 0 + 0 + ... + to 9!/2 times = 0

Sol.9 (D)

$A + B = 2B^T$ .... (1)

$3A + 2B = I$ .... (2)

3 × (1) – (2) gives

$6B^T - B = I$ .... (3)

Taking transpose of both sides of eqn. (3), we have

$6B - B^T = I^T = I$ .... (4)

Now (4) – (3)

$7B - 7B^T = 0$ or $B = B^T$

$\therefore$ From (3)

$6B - B = I$

$\Rightarrow$ $5B = I$ or $B = \frac{I}{5}$

Again from (2) $3A + 2B = I$

$\Rightarrow$ $A = \frac{I}{5}$

Thus $5A + 5B = 2I$

Sol.10 (B)

$$z = \begin{vmatrix} 1 & i & 7+8i \\ -i & 2 & 3+2i \\ 7-8i & 3-2i & 4 \end{vmatrix}$$

$$\bar{z} = \begin{vmatrix} 1 & -i & 7-8i \\ i & 2 & 3-2i \\ 7+8i & 3+2i & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -i & 7+8i \\ -i & 2 & 3+2i \\ 7-8i & 3-2i & 4 \end{vmatrix} \qquad \because |z| = |z^T|$$

$\Rightarrow$ $\bar{z} = z$

$\Rightarrow$ z is purely real

$\Rightarrow$ Im z = 0