Q.1 If p + q + r = 0, then $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} =$

(A) prq $\begin{vmatrix} b & c & a \\ c & a & b \\ b & c & a \end{vmatrix}$ (B) pqr $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

(C) pqr $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (D) None of these

Q.2 Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then for any +ve integer n, the value of $\lim\limits_{n\to\infty} \frac{1}{n^2} A^{-n}$ is-

(A) $\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q.3 If the values of the determinant $\begin{vmatrix} x & y & z \\ 4 & 3 & 2 \\ x41 & y31 & z21 \end{vmatrix}$ is zero where x41, y31 and z21 are three digit numbers, then x, y, z are in-

(A) G.P. (B) A.P.
(C) H.P. (D) No such relation

Q.4 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$ ($\omega \neq 1$, ω is a cube root of unity) then $|A^{-1}|$ is-

(A) $\frac{\omega}{3(1-\omega)}$ (B) $\frac{\omega^2}{3(1-\omega)}$

(C) $\frac{\omega^2}{3(\omega-1)}$ (D) $\frac{\omega}{3(\omega-1)}$

Q.5 If $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3$ is equal to

(A) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (B) $\frac{1}{27}\begin{bmatrix} 1 & -8 \\ 0 & 27 \end{bmatrix}$

(C) $27\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -8 \\ 0 & 1 \end{bmatrix}$

Q.6 The system of equations ax + by = c, a'x + b'y = c' has
(A) a unique solution if ab' – a'b ≠ 0
(B) no solution if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$ (a', b', c' ≠ 0)
(C) infinite solution if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$
(D) All are true

Q.7 For each real number x such that –1 < x < 1, let A(x) be the matrix $(1 - x)^{-1}\begin{bmatrix} 1 & -x \\ -1 & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then
(A) A(z) = A(x) + A(y)
(B) A(z) = A(x) $[A(y)]^{-1}$
(C) A(z) = A(x) A(y)
(D) A(z) = A(x) – A(y)

Q.8 If value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be
(A) 11 (B) 121
(C) 1331 (D) 14641

MATHEMATICS IIT JEE (CLASS TEST - 2) (MATRIX & DETERMINANT) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	⬭	⬭	⬭	⬭	4	⬭	⬭	⬭	⬭	7	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭	5	⬭	⬭	⬭	⬭	8	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭	6	⬭	⬭	⬭	⬭					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	B	B	B	C	A	D	C	D

SOLUTION

Sol.1 (B)

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$$

$= pa(qra^2 - p^2bc) - qb(q^2ac - prb^2) + rc(c^2qp - r^2ab)$

$= pqra^3 - abcp^3 - abcq^3 + pqrb^3 + c^3pqr - r^3abc$

$= pqr(a^3 + b^3 + c^3) - abc(p^3 + q^3 + r^3)$

$= pqr(a^3 + b^3 + c^3 - 3abc) - abc(p^3 + q^3 + r^3 - 3pqr)$

$= pqr(a^3 + b^3 + c^3 - 3abc) - abc(p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$

$= pqr\ (a^3 + b^3 + c^3 - 3abc) - 0$

$= pqr\ (a^3 + b^3 + c^3 - 3abc)$

$$= pqr \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Sol.2 (B)

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\text{adj. } A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \text{ and det } A = 1$$

$$A^{-1} = \frac{\text{adj } A}{\det A} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} B \text{ (say)}$$

$$\therefore \quad B^2 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \Rightarrow B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

$$\therefore \quad \lim_{n\to\infty} \frac{1}{n^2} A^{-n} = \lim_{n\to\infty} \frac{1}{n^2} \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$$

$$= \lim_{n\to\infty} \begin{bmatrix} 1/n^2 & 0 \\ -1/n & 1/n^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Sol.3 (B)

Applying $R_1 \to 100R_1,\ R_2 \to 10R_2$

$$\frac{1}{1000} \begin{vmatrix} 100x & 100y & 100z \\ 10\times4 & 10\times3 & 10\times2 \\ 100x+40+1 & 100y+30+1 & 100z+20+1 \end{vmatrix} = 0$$

Applying $R_1 \to R_3 - R_1 - R_2$

$$\frac{1}{1000} \begin{vmatrix} 100x & 100y & 100z \\ 10\times4 & 10\times3 & 10\times2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & z \\ y & 3 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$\begin{vmatrix} x-y & y-z & z-x \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$= (x - y)\ (y - z) = x + z - 2y = 0$

$\Rightarrow \quad 2y = x + z$

$\Rightarrow \quad x, y, z$ are in A.P.

Sol.4 (C)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$\therefore |A| = (\omega^2 - \omega^4) - 1(\omega - \omega^2) + 1(\omega^2 - \omega)$

$|A| = \omega^2 - \omega^4 - \omega + \omega^2 + \omega^2 - \omega$

$|A| = 3(\omega^2 - \omega)$

$\because \quad |A^{-1}| = |A|^{-1}$

$$= \frac{1}{3(\omega^2 - \omega)} = \frac{\omega^2}{3\omega^2(\omega - 1)\omega} = \frac{\omega^2}{3(\omega - 1)}$$

Sol.5 (A)

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}, \quad |A| = 3, \text{ adj } A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \quad (A^{-1})^3 = \left(\frac{1}{3}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}\right)^3 = \frac{1}{27}\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}^3$$

$$= \frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$$

Sol.6 (D)

The given equation can be written as

$$\begin{bmatrix} a & b \\ a' & b' \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ c' \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ a' & b' \end{bmatrix}^{-1}\begin{bmatrix} c \\ c' \end{bmatrix}$$

(Provided $ab' - a'b \neq 0$)

Thus, $ab' - a'b \neq 0$

$\Rightarrow$ unique solution.

No solution is $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$.

(The equations become inconsistent) and infinite solutions if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

Sol.7 (C)

$$A(z) = A\left(\frac{x+y}{1+xy}\right) = \left[\frac{1+xy}{(1-x)(1-y)}\right]$$

$$\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$$

$\therefore \quad A(x).A(y) = A(z).$

Sol.8 (D)

$\Delta^c = \Delta^{n-1} = \Delta^{3-1} = \Delta^2 = (11)^2 = 121.$

But we have to find the value of the square of the determinant, so required value is

$(121)^2 = 14641.$